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RG flows from super-Liouville theory to critical Ising model

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Abstract

We study an integrable deformation of the super-Liouville theory which generates a RG flows to the critical Ising model as the IR fixed point. This model turns out to be a supersymmetric sinh-Gordon model with spontaneously broken $N = 1$ supersymmetry. The resulting massless goldstino is the only stable on-shell particle which controls the IR behaviours. We propose the exact S -matrix of the goldstino and compare associated thermodynamic Bethe ansatz equations with the quantization conditions derived from the reflection amplitudes of the super-Liouville theory to provide nonperturbative checks for both the (NS) and the (R) sectors. © 2002 Published by Elsevier Science B.V.

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1. Introduction

Integrable quantum field theories defined in two dimensions can be formally written as a UV conformal field theory (CFT) perturbed by some relevant operator [1]. Most well known examples are the unitary minimal CFTs \mathcal{M}_p perturbed by the least relevant field $\Phi_{1,3}$ whose action can be written formally as follows:

$$\mathcal{M}_p^\pm = \mathcal{M}_p + \lambda \Phi_{1,3}. \quad (1)$$

Here, the sign \pm stands for the signature of the coefficient λ . If the coefficient of the perturbation is negative, the perturbed CFT \mathcal{M}_p^- is described by the factorized scattering theory of massive particles called kinks. More interesting is the case of \mathcal{M}_p^+ which is shown to generate RG flows from the UV CFT \mathcal{M}_p to \mathcal{M}_{p-1} . This was first noticed in [2] by perturbative computation for the case of $p \gg 1$ and was proved later by the thermodynamic Bethe ansatz (TBA) based on the S -matrix of the massless kinks [3].

Among these, the RG flows from the tricritical Ising model (TIM) to the critical Ising model draws a particular interest since the TIM is a super CFT while the Ising model is not [4]. An analysis based on the Landau–Ginzburg potential in [5] shows clearly how the RG flow can be understood. The unperturbed

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TIM has a Φ^3 superpotential, which is in components $\bar{\psi}\psi\phi + \frac{1}{8}\phi^4$. The relevant perturbation, $\Phi_{1,3}$, is the top component of the superfield Φ and preserves the supersymmetry. This modifies the superpotential to $\bar{\psi}\psi\phi + \frac{1}{2}(\frac{1}{2}\phi^2 + \lambda)^2$. For $\lambda < 0$ the ground state energy is zero, so supersymmetry is unbroken and both boson and fermion become massive. The S -matrix is nondiagonal and commutes with the supercharges [6].

With the positive coefficient $\lambda > 0$, the superpotential generates nonvanishing ground state energy and the supersymmetry becomes spontaneously broken. The bosonic field becomes massive, but the fermion stays massless and plays the role of goldstino. In the IR limit one can integrate out the massive bosonic field to obtain the effective theory described by the Volkov–Akulov field theory [7]

$$\mathcal{L}_{\text{VA}} = -\frac{1}{2\pi}(\psi\bar{\partial}\psi + \bar{\psi}\partial\bar{\psi}) - g(\psi\partial\psi)(\bar{\psi}\bar{\partial}\bar{\psi}) + \dots, \quad (2)$$

where \dots include higher-dimensional operators.

In this Letter, we propose another RG flow where the supersymmetry is spontaneously broken and the low energy effective action is described by a goldstino. The model is another supersymmetric sinh-Gordon (SShG) model which can be considered as a perturbed super-Liouville field theory (SLFT) [8]. The ordinary SShG model is one of the simplest examples of a $(1+1)$ -dimensional integrable quantum field theory with $N=1$ supersymmetry [9]. A generic Lagrangian including one scalar superfield can be expressed in terms of the component fields as

$$\begin{aligned} \mathcal{L}(\Phi) = & \frac{1}{8\pi}(\partial_a\phi)^2 - \frac{1}{2\pi}(\psi\bar{\partial}\psi + \bar{\psi}\partial\bar{\psi}) \\ & - \frac{i}{4\pi}\psi\bar{\psi}W''(\phi) + \frac{1}{32\pi}[W'(\phi)]^2. \end{aligned} \quad (3)$$

The ordinary SShG model is a particular case of Eq. (3) with the superpotential

$$W(\phi) = -8\pi\mu \cosh(b\phi). \quad (4)$$

The SShG model and its imaginary coupling version ($b \rightarrow i\beta$), the supersymmetric sine-Gordon (SSG) model, are integrable since they can be mapped into an affine Toda theory based on the twisted super-Lie algebra $C^{(2)}(2)$ [10]. This model preserves the supersymmetry and the boson and fermion remain massive. Exact factorized nondiagonal S -matrices have been ob-

tained from the integrability and on-shell supersymmetry in [11,12]. This model is analogous to the TIM with $\lambda < 0$.

Another SShG model, which is our main concern in this Letter, is defined by a slightly different superpotential, namely,

$$W(\phi) = -8\pi\mu \sinh(b\phi). \quad (5)$$

The supersymmetry and integrability are all preserved. If we consider an imaginary coupling $b = i\beta$, the two supersymmetric sine-Gordon models become equivalent since one can shift the scalar field by $\phi \rightarrow \phi + \text{const}$. However, with a real coupling b , the new SShG model shows the RG flows from the UV super-LFT to the critical Ising model at the IR. With the superpotential Eq. (5), the ground state energy does not vanish so that the supersymmetry is spontaneously broken. While the bosonic field ϕ remains massive, the fermion field becomes massless and is identified with goldstino. Supersymmetry prohibits the quantum corrections from generating mass. Meanwhile, the bosonic field ϕ is unstable and decays into the massless fermions. After the massive bosonic field is integrated out, the low energy effective action is described by the Volkov–Akulov action Eq. (2).

Stable on-shell particle states of this model are composed of massless left- and right-moving fermions, ψ_L and ψ_R , respectively. This model can be thought of as a perturbed super-LFT analogous to the perturbed TIM with $\lambda > 0$. The S -matrix between the ψ_L and ψ_R can be conjectured from the unitarity and crossing symmetry as well as a perturbative computation. In this Letter, we propose the S -matrix with the assumption of strong-weak coupling duality.

Non-perturbative confirmation of the conjecture is provided by the TBA analysis. For the cases of perturbed rational CFTs, the UV limit of the TBA provides the central charges and conformal dimensions for the UV CFTs. For the perturbed SLFT, one can extract out an additional information, namely, the reflection amplitudes from the TBA. We analyze the UV behaviour of the TBA equations of the new SShG model with the conjectured S -matrix and compare it with the reflection amplitudes of the super-LFT. Numerical agreement with very high accuracy will establish the correctness of the S -matrix. We also provide the IR analysis of the TBA equations and relate them to the IR action Eq. (2).

2. S-matrix and TBA

Without any mass degeneracy, the S -matrix of the new SShG model is diagonal. The only interaction term, $\psi_L \psi_R \sinh(b\phi)$, if expressed with chiral fermions ψ_L and ψ_R , gives trivial scattering between the two ψ_L 's (and two ψ_R 's), i.e.,

$$S_{LL}(\theta) = S_{RR}(\theta) = -1. \tag{6}$$

Nontrivial S -matrix arises between a ψ_L and a ψ_R . Since the particle is massless, the scattering amplitude satisfies the crossing-unitarity relation,

$$S_{LR}(\theta)S_{LR}(\theta + i\pi) = 1. \tag{7}$$

This equation is solved by the CDD factor

$$S_{LR}(\theta) = \frac{\sinh \theta - i \sin \pi B}{\sinh \theta + i \sin \pi B}, \tag{8}$$

where we fix the rapidity by choosing a scale M in such a way that the energy–momentum is given by (for the ψ_R) $E = P = \frac{M}{2}e^\theta$. Apparently Eq. (8) is not the unique CDD choice. It is the minimal CDD factor which contains the proper resonance pole in the s - and u -channels with the resonance mass $m^2 = M^2 e^{-i\pi B}$.

Without any bootstrap procedure, we cannot fix the location of the resonance pole. Our conjecture for the parameter B is

$$B(b) = \frac{b^2}{1 + b^2}. \tag{9}$$

This is consistent with perturbation theory up to the second order and preserves the duality $b \rightarrow 1/b$ enjoyed by the ordinary SShG model. The duality has root in its UV CFT, namely the super-LFT which is dual. Since the new SShG model can be also considered as a perturbed super-LFT, it is plausible to assume the duality in our case. Subsequently, we will provide nonperturbative confirmation of the S -matrix.

For this purpose, we compute the effective central charge of the SShG model using the TBA analysis. It is straightforward to write down the TBA equations from the S -matrix.

$$\epsilon_L(\theta) = \frac{1}{2}MR e^\theta - \int_{-\infty}^{\infty} \varphi(\theta - \theta') \ln(1 + \eta e^{-\epsilon_R(\theta')}) \frac{d\theta'}{2\pi}, \tag{10}$$

$$\epsilon_R(\theta) = \frac{1}{2}MR e^{-\theta} - \int_{-\infty}^{\infty} \varphi(\theta - \theta') \ln(1 + \eta e^{-\epsilon_L(\theta')}) \frac{d\theta'}{2\pi}, \tag{11}$$

where the parameter η is either $+1$ for the Neveu–Schwarz (NS) sector or -1 for the Ramond (R) sector and the kernel, the logarithmic derivative of the S -matrix, is given by

$$\varphi(\theta) = \frac{4 \sin \pi B \cosh \theta}{\cosh 2\theta - \cos 2\pi B}. \tag{12}$$

The effective central charge is given by

$$c_{\text{eff}}(R) = \frac{3MR}{2\pi^2} \int_{-\infty}^{\infty} [e^\theta \ln(1 + \eta e^{-\epsilon_L(\theta)}) + e^{-\theta} \ln(1 + \eta e^{-\epsilon_R(\theta)})] d\theta. \tag{13}$$

This TBA equation can be solved analytically in the UV region $MR \ll 1$. Here, the $c_{\text{eff}}(R)$ has logarithmic corrections of $1/\log(MR)^n$ as leading contributions and subleading power corrections. In particular, the R^2 term in $c_{\text{eff}}(R)$ can be interpreted as the vacuum energy contribution. The analysis gives

$$\mathcal{E}_0 = \frac{M^2}{8 \sin(\pi B)} \tag{14}$$

which is the same as that of the sinh-Gordon model. This result is somewhat expected since the vacuum expectation value of the interacting potential can be determined by the (NS) reflection amplitude of the $N = 1$ super-LFT, which is the same as that of the LFT. To compare the TBA result with the reflection amplitude, one needs a relation between the dimensionful parameter μ and the mass scale parameter M for the SShG model. We conjecture that this is the same as that of the ordinary SShG model given in [13],

$$\frac{\pi}{2} \mu b^2 \gamma\left(\frac{1 + b^2}{2}\right) = \left[\frac{M}{8} \frac{\pi B}{\sin \pi B}\right]^{1+b^2} \tag{15}$$

with $\gamma(x) = \Gamma(x)/\Gamma(1 - x)$. These conjectures will be confirmed by numerical analysis of the TBA equations in Section 4.

3. Reflection amplitudes and quantization condition

The SShG model can be considered as a perturbed super-LFT whose Lagrangian is given by

$$\mathcal{L}_{\text{SL}} = \frac{1}{8\pi}(\partial_a\phi)^2 - \frac{1}{2\pi}(\bar{\psi}\partial\bar{\psi} + \psi\bar{\partial}\psi) + i\mu b^2\psi\bar{\psi}e^{b\phi} + \frac{\pi\mu^2b^2}{2}e^{2b\phi}. \quad (16)$$

With the background charge Q

$$Q = b + 1/b. \quad (17)$$

This model is a CFT with the central charge

$$c_{\text{SL}} = \frac{3}{2}(1 + 2Q^2) \quad (18)$$

and primary fields in the (NS) and (R) sectors. A (NS) primary field $e^{\alpha\phi}$ has dimension

$$\Delta_\alpha = \frac{1}{2}\alpha(Q - \alpha) \quad (19)$$

and becomes degenerate with $e^{(Q-\alpha)\phi}$. The two-point functions of the primary fields give the reflection amplitudes [14,15]. For the (NS) field, it is given by

$$S_{\text{NS}}(P) = -\left(\frac{\pi\mu}{2}\gamma\left(\frac{1+b^2}{2}\right)\right)^{-2iP/b} \times \frac{\Gamma(1+iPb)\Gamma(1+\frac{iP}{b})}{\Gamma(1-iPb)\Gamma(1-\frac{iP}{b})}. \quad (20)$$

Similarly, for a (R) field $\sigma^{(\epsilon)}e^{\alpha\phi}$ the reflection amplitude is given by

$$S_{\text{R}}(P) = \left(\frac{\pi\mu}{2}\gamma\left(\frac{1+b^2}{2}\right)\right)^{-2iP/b} \times \frac{\Gamma(\frac{1}{2}+iPb)\Gamma(\frac{1}{2}+\frac{iP}{b})}{\Gamma(\frac{1}{2}-iPb)\Gamma(\frac{1}{2}-\frac{iP}{b})}. \quad (21)$$

To derive quantization conditions, one can consider the super-LFT acting on the space of states

$$A_0 = \mathcal{L}_2(-\infty < \phi_0 < \infty, \psi_0) \otimes \mathcal{F}, \quad (22)$$

where the fermionic zero-mode appears only for the (R) sector and \mathcal{F} is the Fock space of bosonic and fermionic oscillators. The appearance of bosonic and fermionic zero-modes in Eq. (22) is well known from the super-CFT results. In the (NS) sector, there is no

fermionic zero-mode since the fermion field satisfies the anti-periodic boundary condition while it appears in the (R) sector with periodic one. The primary state v_P can be expressed by a wave functional $\Psi_{v_P}[\phi(x_1)]$ which can be expanded in the asymptotic limit $\phi_0 \rightarrow \infty$ as

$$\Psi_{v_P}[\phi(x_1)] \sim e^{iP\phi_0} + S(P)e^{-iP\phi_0}. \quad (23)$$

The amplitude $S(P)$ is either $S_{\text{NS}}(P)$ or $S_{\text{R}}(P)$ depending on the sector.

The ordinary SShG model defined by Eq. (4) can be considered as the super-LFT (16) perturbed by

$$\Phi_{\text{pert}} = i\mu b^2\psi\bar{\psi}e^{-b\phi} + \frac{\pi\mu^2b^2}{2}e^{-2b\phi}. \quad (24)$$

In the wave functional interpretation, the perturbing potential provides another potential wall which confines the wave functional. This leads to the quantization condition for the momentum and the energy of the system in the limit that the size of the cylinder R goes to 0. The quantization condition and comparison with the TBA based on the nondiagonal S -matrix of the ordinary SShG model have been worked out in [16].

The new SShG model, being considered as another perturbed super-LFT by

$$\Phi_{\text{pert}} = -i\mu b^2\psi\bar{\psi}e^{-b\phi} + \frac{\pi\mu^2b^2}{2}e^{-2b\phi}, \quad (25)$$

can be analyzed in the same way. One can obtain the quantization condition of P for the (NS) sector,

$$\delta_{\text{NS}}(P) = \pi + 2QP \ln \frac{R}{2\pi}, \quad (26)$$

where $\delta_{\text{NS}}(P)$ is the phase factor of (NS) reflection amplitudes. Similarly, the quantization condition for the (R) sector becomes

$$\delta_{\text{R}}(P) = \frac{\pi}{2} + 2QP \ln \frac{R}{2\pi}. \quad (27)$$

Notice that the main difference arises from the extra -1 factor in front of the perturbing potential in Eq. (25). Both conditions are invariant under $b \rightarrow 1/b$.

In terms of the quantized momentum P , the effective central charge is given by

$$c_{\text{eff}}(R) = \begin{cases} \frac{3}{2} - 12P^2 + \frac{6}{\pi}R^2\mathcal{E}_0 & \text{(NS),} \\ -12P^2 + \frac{6}{\pi}R^2\mathcal{E}_0 & \text{(R),} \end{cases} \quad (28)$$

where we added the vacuum energy \mathcal{E}_0 to compare the same ground-state energy.

This quantization condition can be solved iteratively by expanding $\delta(P)$ in powers of P and be compared with the numerical TBA solutions

$$\delta_K(P) = \delta_1^K P + \delta_3^K P^3 + \delta_5^K P^5 + \dots, \tag{29}$$

where K stands for either NS or R. Explicitly, the coefficients for the (NS) are given by

$$\begin{aligned} \delta_1^{\text{NS}} &= -2 \left\{ \frac{1}{b} \ln \left[\frac{\pi \mu}{2} \gamma \left(\frac{1+b^2}{2} \right) \right] + \gamma_E Q \right\}, \\ \delta_3^{\text{NS}} &= \frac{2}{3} \zeta(3) \left(b^3 + \frac{1}{b^3} \right), \\ \delta_5^{\text{NS}} &= -\frac{2}{5} \zeta(5) \left(b^5 + \frac{1}{b^5} \right), \end{aligned} \tag{30}$$

and, for the (R),

$$\begin{aligned} \delta_1^{\text{R}} &= -2 \left\{ \frac{1}{b} \ln \left[\frac{\pi \mu}{2} \gamma \left(\frac{1+b^2}{2} \right) \right] + (\gamma_E + 2 \ln 2) Q \right\}, \\ \delta_3^{\text{R}} &= -\frac{1}{3} \psi^{(2)} \left(\frac{1}{2} \right) \left(b^3 + \frac{1}{b^3} \right), \\ \delta_5^{\text{R}} &= \frac{1}{60} \psi^{(4)} \left(\frac{1}{2} \right) \left(b^5 + \frac{1}{b^5} \right). \end{aligned} \tag{31}$$

4. TBA analysis

To derive the coefficients δ 's from the TBA equations, we derive the momentum P as a function of R from the scaling function $c_{\text{eff}}(R)$ and compare with the quantization conditions to determine the coefficients. In Tables 1 and 2, we show the first three coefficients in the expansion in powers of P obtained by numerical analysis and compare with the corresponding coefficients from the reflection amplitudes Eqs. (20) and (21) with $M = 2$.

Our numerical analysis shows the consistency of the TBA equations along with such conjectures as the scattering amplitude, the $M-\mu$ relation as well as the reflection amplitudes of the super-LFT Eqs. (20) and (21). Also one can see from Fig. 1 that the vacuum energy Eq. (14) improves the agreement with the TBA result much better than without it (dotted lines) up to the range of $0.1 < MR < 1$. This observation provides the validity of the conjectured vacuum energy.

We want to point out that a similar analysis for the ordinary SShG model in [16] could not give any non-perturbative confirmation for the (R) reflection amplitude as well as the vacuum energy since the scaling function for the (R) sector and the vacuum energy vanish identically. The new SShG model provides the unique ‘‘experiment’’ for these quantities.

Table 1
First three coefficients of $\delta^{\text{NS(TBA)}}$ in the expansion in powers of P obtained by numerical analysis in comparison with the corresponding $\delta^{\text{NS(RA)}}$

B	$\delta_1^{\text{NS(TBA)}}$	$\delta_1^{\text{NS(RA)}}$	$\delta_3^{\text{NS(TBA)}}$	$\delta_3^{\text{NS(RA)}}$	$\delta_5^{\text{NS(TBA)}}$	$\delta_5^{\text{NS(RA)}}$
0.3	0.276167	0.276167	3.08111	3.08111	-3.49936	-3.49933
0.35	0.823499	0.823499	2.34480	2.34480	-2.03777	-2.03774
0.4	1.17240	1.17240	1.90842	1.90842	-1.29352	-1.29349
0.45	1.36725	1.36725	1.67590	1.67590	-0.936165	-0.936139
0.5	1.42998	1.42998	1.60274	1.60274	-0.829567	-0.829542

Table 2
First three coefficients of $\delta^{\text{R(TBA)}}$ in the expansion in powers of P obtained by numerical analysis in comparison with the corresponding $\delta^{\text{R(RA)}}$

B	$\delta_1^{\text{R(TBA)}}$	$\delta_1^{\text{R(RA)}}$	$\delta_3^{\text{R(TBA)}}$	$\delta_3^{\text{R(RA)}}$	$\delta_5^{\text{R(TBA)}}$	$\delta_5^{\text{R(RA)}}$
0.3	-5.77412	-5.77412	21.5677	21.5677	-108.480	-108.479
0.35	-4.98943	-4.98943	16.4136	16.4136	-63.1708	-63.1699
0.4	-4.48712	-4.48712	13.3590	13.3590	-40.0991	-40.0982
0.45	-4.20586	-4.20586	11.7313	11.7313	-29.0212	-29.0203
0.5	-4.11519	-4.11519	11.2192	11.2192	-25.7167	-25.7158

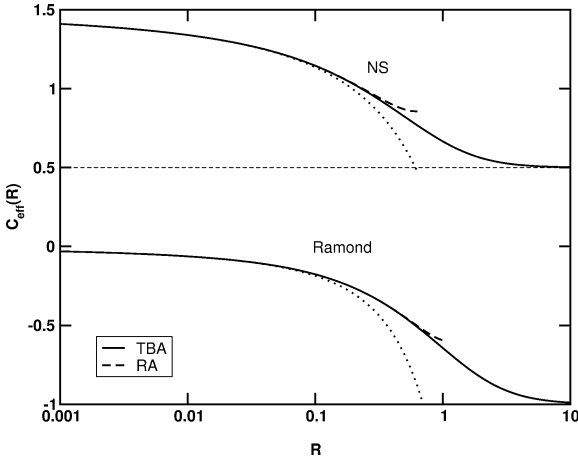


Fig. 1. Plot of c_{eff} for the (NS) and (R) sectors at $B = 0.5$.

As suggested by numerical analysis, the SShG model flows into the Ising model in the IR limit, $R \rightarrow \infty$. The effective central charge in this limit is given by $c_{\text{NS}} = 1/2$ for the (NS) sector and $c_{\text{R}} = -1$ for the (R) sector where the Ramond vacuum with the conformal dimension $1/16$ is contributed.

In the IR limit $MR \ll 1$, the main contributions comes from the rapidity regions where pseudo energy $\epsilon(\theta) \leq 1$. The asymptotic expansion can be obtained straightforwardly for the (NS) and (R) sectors as follows:

$$c_{\text{NS}} = \frac{1}{2} + \frac{1}{4}t + \frac{1}{4}t^2 + \left(\frac{5}{16} + \frac{147\pi^2}{400} \frac{(2 \cos 2\pi B + 1)}{\sin^2 \pi B} \right) t^3 + O(t^4), \quad (32)$$

$$c_{\text{R}} = -1 + t - 2t^2 + \left(5 + \frac{4\pi}{15} + \frac{12\pi^2}{25} \frac{(2 \cos 2\pi B + 1)}{\sin^2 \pi B} \right) t^3 + O(t^4), \quad (33)$$

where

$$t = \frac{4\pi \sin \pi B}{3(MR)^2}. \quad (34)$$

This IR behaviour can be described in terms of the Ising model with $T\bar{T}$ perturbation, Eq. (2). The perturbation contributes to c_{eff}

$$c_{\text{eff}} = c - 12 \left(\frac{c}{24} \right)^2 \alpha + 12 \left(\frac{c}{24} \right)^3 \alpha^2 + O(\alpha^3) \quad (35)$$

with $\alpha = -32\pi^3 g/R^2$ where g is the coupling coefficient in Eq. (2). Higher order term is ambiguous due to the UV regularization. Two results for the (NS) sector, Eqs. (32) and (35), are consistent up to order α^2 and t^2 if we identify $c = 1/2$ and $g = 2 \sin \pi B/\pi^2 M^2$. For the (R) sector, Eq. (33) is consistent with Eq. (35) when $c = -1$ and g is the same as before. Notice that the RG flow from the TIM to the Ising model is described by Eq. (2) with $g = 1/\pi^2 M^2$.

In summary, we have considered the SShG model with spontaneously broken supersymmetry with a massless goldstino which generates the RG flows from the super-LFT to the Ising model for the (R) and (NS) sectors. We propose a set of conjectures such as the S -matrix, $M-\mu$ relation, and the vacuum energy. These conjectures are eventually justified by the independently driven effective central charge based on reflection amplitudes of the super-LFT.

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