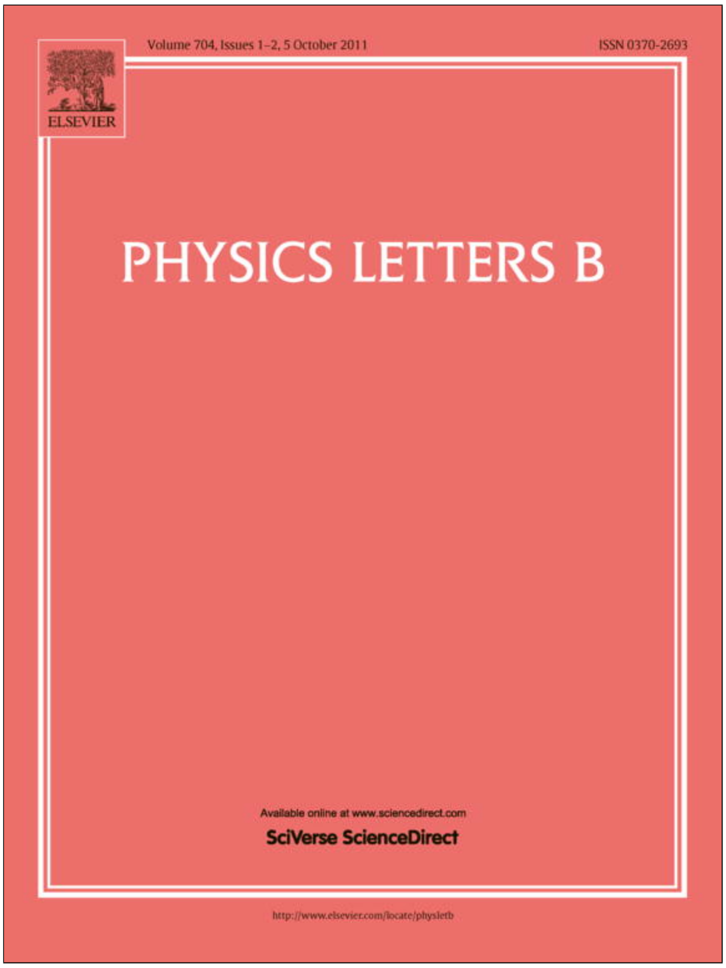
Volume 704, Issues 1–2, 5 October 2011
ISSN 0370-2693
ELSEVIER
PHYSICS LETTERS B
Available online at www.sciencedirect.com
SciVerse ScienceDirect
http://www.elsevier.com/locate/physletb

Physics Letters B 704 (2011) 10–14

Contents lists available at SciVerse ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb

From asymptotic safety to dark energy

Changrim Ahn [a,*], Chanju Kim [a], Eric V. Linder [a,b]

[a] Institute for the Early Universe WCU and Department of Physics, Ewha Womans University, Seoul, Republic of Korea
[b] Berkeley Lab and University of California, Berkeley, USA

ARTICLE INFO

Article history:
Received 13 June 2011
Accepted 29 August 2011
Available online 2 September 2011
Editor: M. Trodden

ABSTRACT

We consider renormalization group flow applied to the cosmological dynamical equations. A consistency condition arising from energy–momentum conservation links the flow parameters to the cosmological evolution, restricting possible behaviors. Three classes of cosmological fixed points for dark energy plus a barotropic fluid are found: a dark energy dominated universe, which can be either accelerating or decelerating depending on the RG flow parameters, a barotropic dominated universe where dark energy fades away, and solutions where the gravitational and potential couplings cease to flow. If the IR fixed point coincides with the asymptotically safe UV fixed point then the dark energy pressure vanishes in the first class, while (only) in the de Sitter limit of the third class the RG cutoff scale becomes the Hubble scale.

1. Introduction

Cosmic acceleration may be due to a scalar field arising from high energy physics. This creates two puzzles: if the physics is set at the Planck scale, or similarly high energy, why is the magnitude of the energy density, basically the amplitude of the scalar field potential, of order $(10^{-3}\ \text{eV})^4$ today, and why doesn't the potential amplitude and shape receive strong corrections from couplings in the high energy universe?

A useful and efficient way of analyzing quantum effects on the low energy scale physics is the renormalization group (RG) [1]. An effective theory is obtained by integrating out the quantum fluctuations with higher energy scales than a certain cutoff scale. It contains a number of parameters that run along with the cutoff scale, called the RG flows. One can then incorporate the quantum effects using classical equations of motion from the effective action. The main problem in applying the RG approach to cosmology is that we do not know the complete quantum gravity theory that governs the UV (Planck) scale physics. Asymptotically safe gravity [2] is an idea that the quantum gravity is described by a finite number of parameters which approach nontrivial fixed points in the UV scale limit. This provides a conceptual framework to link the UV physics with the low energy effective theory that describes physics at much later time scale.

We explore here the cosmological late time effects from renormalization group flow. This differs from the application of asymptotic safety criteria in the UV (see [3] for a review) in that we focus on the IR behavior of the field and its effects on dark energy and the cosmological expansion. We look for cosmological fixed points to the coupled dynamical equations including RG effects, which may or may not correspond to fixed points of the RG flow.

The low energy effective action can, in principle, be obtained from the RG equation. This is however a highly nontrivial functional differential equation with respect to the RG scale k [4] that is virtually impossible to solve exactly. As a simple approximation, we (in agreement with much of the literature) shall adopt the Einstein–Hilbert truncation [5] in the gravity part by neglecting higher derivative terms. In the matter part, the kinetic term of the scalar field is taken to be canonical (i.e. no running since there is no coupling parameter) while the potential is allowed to vary as the RG scale k changes. In this approximation, the practical effect of RG flow is then an evolution of the gravitational coupling G_k (generalizing Newton's constant) and the scalar field potential $V_k(\phi)$. In particular the equations of motion will take the same form as the classical ones.

In the application of the RG to cosmology, the RG evolution governed by the RG cutoff k is then related to the cosmological evolution in time t. In the literature the cutoff k is usually assumed to be proportional to $1/t$ on the physical ground that fluctuations smaller than $1/t$ do not play any role, thus providing the IR cutoff [6,7]. Another choice would be for example $k \sim H(t)$, the Hubble parameter at t. Note that $H \sim 1/t$ in general so this is quite similar. See [8–10] and references therein for this and other cutoffs, some inspired by holography, applied to cosmology.

However, the truncation of the low energy effective action to the Einstein–Hilbert form already restricts the type of influence of

* Corresponding author.
E-mail address: tejeho@gmail.com (C. Ahn).

0370-2693/$ – see front matter
doi:10.1016/j.physletb.2011.08.075

the cutoff scale on the cosmology. We will see that a constraint among RG parameters emerges for consistency of the approximation. The constraint has been discussed in previous works with a perfect fluid [6,7]. In this Letter we will consider the gravity with barotropic fluids and a scalar field and derive the constraint and its consequences.

In deriving the modifications of the Einstein field equations from the Einstein–Hilbert action with non-constant couplings, three approaches can be taken, depending on the interpretation used. One method [11] is to treat the evolution of the couplings as due to a dynamical variable, say a field ϕ. This is basically equivalent to a treatment like $F(\phi)R$ in the case of gravitational coupling, and leads to an extended quintessence type of scalar–tensor theory [12–16], with similarities to induced gravity [17–19].

A second method is to keep the couplings as nondynamical during the variation of the action with respect to the metric, and further assume that continuity equation for each energy–momentum component individually is unaffected. That is, require the part of the covariant derivatives with respect to spacetime coordinates and that part coming from a partial derivative of the renormalization scale k with respect to spacetime coordinates to vanish separately. This was the approach recently taken by [20] (also see [21]). Third, one can keep the couplings nondynamical in the action and require only the Bianchi identity to hold with respect to the total covariant derivatives, simultaneously accounting for the spacetime dependence and the flow of the couplings under the renormalization group. This is the approach we take here, and its results, for example that the flow converges in de Sitter space, indicate that it is of interest in its physical consequences.

In Section 2 we derive the effective dark energy contributions to the Friedmann equations and continuity equations, and the necessary consistency condition between them. We evaluate the system of dynamical equations in Section 3, finding the cosmological fixed points. The relation of the RG cutoff scale to cosmology is addressed in Section 4 and we conclude in Section 5.

2. Cosmology with RG flow

We assume that the universe is described by Einstein gravity with matter (or other barotropic fluids) and a canonical, minimally coupled scalar field. In order to incorporate the quantum effects, we consider the truncated RG flow leading to the Einstein–Hilbert action as explained above. The couplings, including the gravitational coupling and the scalar field potential, will be assumed to run with scale. Because the field equations arise from variation of the action with respect to the metric, and there is no explicit dependence of the couplings on the metric, the form of the field equations will be unaltered. In particular, for a homogeneous and isotropic universe the standard form of the Friedmann equations for the expansion rate H and the acceleration $\ddot{a}$ (or $\dot{H}$) will be preserved.

The evolution equations are

$$H^2 = \frac{8\pi G_k}{3}\rho_k, \tag{1}$$

$$\dot{H} = -4\pi G_k(\rho_k + p_k), \tag{2}$$

where $H = \dot{a}/a$, an overdot represents a time derivative, ρ_k represents the total energy density including all components (e.g. matter, scalar field, etc.), and p_k is the total pressure. We show subscripts k on the gravitational coupling (generalization of Newton's constant) G_k and quantities involving the scalar field coupling, i.e. the potential, to remind that these may flow with the RG scale k.

Note that in a scalar–tensor theory, where the time variation of the gravitational coupling arises from a dynamical field, the form of the Friedmann equations *will* be modified. Extra terms involving $\dot{G}$ and $\ddot{G}$ will appear.

One also has the Bianchi identity, involving the covariant derivative of each side of the Einstein field equation. This gives

$$\begin{aligned} 0 &= \left(G_k T_k^{\mu\nu}\right)_{;\nu} \\ &= \left(G_k T_k^{\mu\nu}\right)_{,\nu} + G_k \Gamma^{\mu}{}_{\alpha\nu} T_k^{\alpha\nu} + G_k \Gamma^{\nu}{}_{\alpha\nu} T_k^{\mu\alpha}. \end{aligned} \tag{3}$$

For the $\mu = 0$ equation in a Friedmann–Robertson–Walker cosmology one gets

$$(G_k\rho_k)_{,0} + 3HG_k(\rho + p_k) = 0. \tag{4}$$

Finally, one must take into account that the time derivative involves a piece from the possible time variation of the RG scale k to find the continuity equation

$$\frac{\partial\rho_k}{\partial t} = -3H\rho_k\left[1 + \frac{p_k}{\rho_k} + \frac{1}{3}\frac{d\ln k}{dN}\frac{\partial\ln(G_k\rho_k)}{\partial\ln k}\right], \tag{5}$$

where $N = \ln a$. Checking against $\partial\rho_k/\partial t$ derived by differentiating Eq. (1) and substituting into Eq. (2) gives agreement. That is, preservation of the form of the Friedmann equations necessarily implies a modification to the continuity equation due to the flow of the RG scale.

For the matter component, the continuity equation reads

$$\partial_t\rho_m = -3H\rho_m\left[1 + \frac{1}{3}\frac{\partial\ln G_k}{\partial\ln k}\frac{d\ln k}{dN}\right]. \tag{6}$$

Note that the evolution is altered from the usual behavior due to the flow of the gravitational coupling. For the dark energy component, the evolution is

$$\begin{aligned} \partial_t\rho_{de} &= -3H(\rho_{de} + p_{de}) \\ &\quad - H\frac{d\ln k}{dN}\left(\frac{\partial\ln G_k}{\partial\ln k}\rho_{de} + \frac{\partial\ln V_k}{\partial\ln k}V_k\right). \end{aligned} \tag{7}$$

Note that only the coupling coefficients within the potential change under the RG flow, and the kinetic term and matter density are unchanged. For the total density, the continuity equation is

$$\begin{aligned} \partial_t\rho_k &= -3H(\rho_k + p_k) \\ &\quad - H\frac{d\ln k}{dN}\left(\frac{\partial\ln G_k}{\partial\ln k}\rho_k + \frac{\partial\ln V_k}{\partial\ln k}V_k\right). \end{aligned} \tag{8}$$

We can verify that the total density equation is indeed consistent with the sum of the individual components, as another check on the system of equations.

However, a crucial further condition is that the variation of the action with respect to the field ϕ gives an unaltered Klein–Gordon equation, since there is no explicit k dependence of ϕ. This field equation must be consistent with the continuity equation we derived. Introducing the RG flow parameters (also called the anomalous dimensions) arising from the flow of the effective action,

$$\eta \equiv \frac{\partial\ln G_k}{\partial\ln k}, \tag{9}$$

$$\nu \equiv \frac{\partial\ln V_k}{\partial\ln k}, \tag{10}$$

and taking the derivative of $\rho_{de} = (1/2)\dot{\phi}^2 + V_k$ we find

$$\begin{aligned} \partial_t\rho_{de} &= \dot{\phi}\ddot{\phi} + \frac{\partial V}{\partial\phi}\dot{\phi} \\ &= -3H\dot{\phi}^2 - H\eta\frac{d\ln k}{dN}\left(\frac{1}{2}\dot{\phi}^2 + V_k\right) - H\nu\frac{d\ln k}{dN}V_k, \end{aligned} \tag{11}$$

where the second line comes from the continuity equation (7) for the dark energy. Only the first term of the second line appears in the Klein–Gordon equation, though, so consistency of the theory requires that the terms proportional to $d\ln k/dN$ must vanish.

This condition leads to two possibilities: either $d\ln k/dN = 0$ for all time, in which case there is no relation between cosmological evolution and renormalization group flow, or

$$0 = \frac{1}{2}\dot{\phi}^2\eta + V_k(\eta+\nu). \tag{12}$$

This is a crucial point because it restricts arbitrary behavior of the RG flow cosmology for the truncated, i.e. Einstein–Hilbert action.

In summary, the gravitational field equations, Bianchi identity, and field evolution equation give a consistent framework within which to treat renormalization group flow and cosmological dynamics together when Eq. (12) is applied.

3. System of dynamical equations

To evaluate the cosmological evolution we can rewrite the cosmological evolution equations in the standard way (see, e.g., [22]) as a coupled system of equations for the dynamics. We use the dynamical variables

$$x^2 = \frac{\kappa^2\dot{\phi}^2}{6H^2}, \tag{13}$$

$$y^2 = \frac{\kappa^2 V_k}{3H^2}, \tag{14}$$

where $\kappa^2 = 8\pi G_k$. The system of equations is

$$\frac{dx}{dN} = -3x\left(1-x^2\right) + \sqrt{\frac{3}{2}}\lambda y^2 + \frac{3}{2}\Sigma x + \frac{\eta}{2}x\frac{d\ln k}{dN}, \tag{15}$$

$$\frac{dy}{dN} = -\sqrt{\frac{3}{2}}\lambda xy + 3x^2 y + \frac{3}{2}\Sigma y + \frac{\eta+\nu}{2}y\frac{d\ln k}{dN}, \tag{16}$$

where the logarithmic potential slope

$$\lambda \equiv -\frac{1}{\kappa V_k}\frac{dV_k}{d\phi}, \tag{17}$$

and $\Sigma = \sum_{i\neq de}(1+w_i)\Omega_i(a)$. The sum includes all barotropic fluids present such as matter or radiation, but the scalar field component is treated separately. Here Ω_i is the dimensionless energy density in barotropic component i and w_i is its equation of state parameter or pressure to density ratio (e.g. 0 for matter, 1/3 for radiation).

Thus the time dependence of the RG cutoff parameter $k(N)$ will be an important element in the cosmological dynamics.

Cosmological quantities of interest will be the effective dark energy density and its equation of state defined through $d\ln\rho_{de}/dN = -3(1+w)$,

$$\Omega_{de} = x^2 + y^2, \tag{18}$$

$$w = \frac{x^2-y^2}{x^2+y^2}. \tag{19}$$

Note that the influence of the RG flow is implicit in the behavior of x and y; the correction term in Eq. (7) vanishes due to the consistency condition and so the scale k does not explicitly appear.

In the case where $d\ln k/dN = 0$ for all time, there is no RG flow and the standard cosmological dynamics results apply. We therefore do not consider this case further. The necessary consistency condition of Eq. (12) then becomes in terms of the dynamical variables,

$$y^2 = \frac{-\eta}{\eta+\nu}x^2. \tag{20}$$

(Note that we expect η to be negative.) Applying this to the cosmological quantities gives

$$\Omega_{de} = x^2\frac{\nu}{\eta+\nu}, \tag{21}$$

$$w = \frac{2\eta+\nu}{\nu}. \tag{22}$$

We will be particularly interested in fixed point solutions of the dynamics, asymptotic behaviors that are insensitive to initial conditions and can serve as cosmological, and possibly RG flow, attractors.

In searching for such solutions, first consider $y = 0$. Then the solutions are either $x = 0$, which implies $\Omega_{de} = 0$, i.e. the vanishing of dark energy, or $d\ln k/dN = 0$, $x = 1$ and $\Sigma = 0$, i.e. complete dark energy domination with $\Omega_{de} = 1$ but $w = 1$ so this is a kinetic energy dominated solution that decelerates the expansion. Both of these are standard cosmology solutions in the absence of RG flow, since asymptotically $d\ln k/dN = 0$, i.e. the RG flow freezes. However, the trajectory to reach the fixed point in general differs in the RG cosmology.

If $x = 0$ the fixed points are $y = 0$ as already considered, or $d\ln k/dN = 0$ with $\lambda = 0$ (as in a runaway, e.g. inverse power law potential). This solution is dark energy dominated with $\Omega_{de} = 1$ and $w = -1$, so this is a potential energy dominated case that accelerates the expansion, ending in a de Sitter state. Again, this asymptotically agrees with a standard cosmology fixed point.

In the case where asymptotically $d\ln k/dN = 0$ (but $x \neq 0 \neq y$), the critical points are

$$x_{c1}^2 = \frac{\lambda^2}{6}, \qquad x_{c2}^2 = \frac{3}{2}\frac{(1+w_b)^2}{\lambda^2},$$
$$y_{c1}^2 = 1-\frac{\lambda^2}{6}, \qquad y_{c2}^2 = \frac{3}{2}\frac{1-w_b^2}{\lambda^2},$$
$$\Omega_{de,c1} = 1, \qquad \Omega_{de,c1} = \frac{3(1+w_b)}{\lambda^2},$$
$$w_{c1} = -1+\frac{\lambda^2}{3}, \qquad w_{c2} = w_b. \tag{23}$$

The first critical point is dark energy dominated, with an equation of state depending on the value of λ. If $\lambda = 0$ asymptotically, then the dynamics approaches a de Sitter state. A stable fixed point only exists for $\lambda^2 < 3$. The second critical point is a scaling solution where dark energy and the least positive equation of state barotropic component have densities in a constant ratio. Since w is equal to the equation of state of the barotropic component w_b then this cannot give acceleration unless one already had an accelerating barotropic component.

Going beyond these cases, there is only one general solution since y is not independent of x due to the consistency requirement of Eq. (20),

$$x_{c3}^2 = 1+\frac{\eta}{\nu},$$
$$y_{c3}^2 = \frac{-\eta}{\nu},$$
$$\Omega_{de,c3} = 1,$$
$$w_{c3} = 1+\frac{2\eta}{\nu}. \tag{24}$$

This is a dark energy dominated solution with the possibility of a variety of equations of state, depending on the specific renormalization group theory. In particular, the case with $\eta = -2$, $\nu = 4$

corresponding to the asymptotically safe UV fixed point for the RG flow (not the cosmology) gives $w = 0$ asymptotically, i.e. the cosmological dynamics behaves asymptotically in the future like a matter dominated universe.

The RG scale parameter here evolves at the cosmological fixed point as

$$\frac{d\ln k}{dN} = \frac{2}{\nu}\left[-3 + \lambda\sqrt{\frac{3}{2}\frac{\nu}{\eta+\nu}}\right]. \tag{25}$$

Recall that the matter in general does not have an equation of state of zero, but rather

$$w_m = \frac{\eta}{3}\frac{d\ln k}{dN}. \tag{26}$$

If one wanted $w_m = 0$ then one must shut off the evolution of the scale k through choosing $\lambda^2 = 6(1+\eta/\nu)$. However, the matter is irrelevant asymptotically in the dark energy dominated solution above. (Also see the next section for further discussion.)

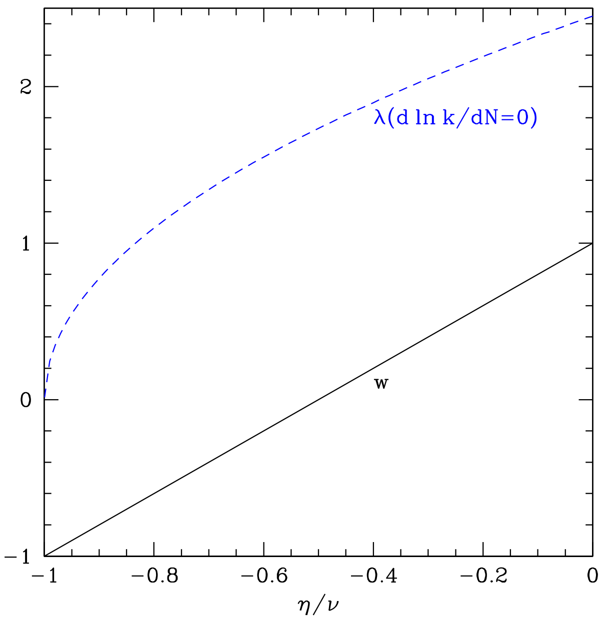

Fig. 1. The ratio η/ν of the anomalous dimensions of the RG flow variables determines the cosmological fixed point for the third critical point. The solid black curve shows the dark energy equation of state dominating the future cosmic expansion, while the blue dashed curve shows the value of λ for which the RG flow freezes. Runaway potentials such as inverse power laws give $\lambda = 0$ and so $w = -1$. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this Letter.)

Fig. 1 shows the dependence on η/ν of the fixed point values for the dark energy equation of state w in this case, and the potential slope λ needed to freeze the RG flow.

In order to end in a de Sitter state in this case, one needs $\eta = -\nu$. Going back to the original consistency condition on the Klein–Gordon equation (11), this requires $\eta x^2(d\ln k/dN) = 0$. So either we reduce to the previous $d\ln k/dN = 0$ solution that gave de Sitter behavior, or we take $\eta = 0 = \nu$, which requires the previous $x = 0$, $\lambda = 0$ solution. Thus the list of cases giving $w = -1$ is complete.

The cosmological attractor solutions are summarized in Table 1. The asymptotic de Sitter solutions with $\lambda = 0$ can be achieved by a runaway potential such as an inverse power law $V \sim \phi^{-n}$ [23], with the field ϕ rolling to the zero potential minimum, without the need for an explicit cosmological constant.

4. Relation of cutoff scale to Hubble scale

Finally, let us examine the issue of the dependence of the renormalization cutoff scale k on the Hubble scale H. Taking the derivative of Eq. (1) with respect to $\ln k$ one obtains (also see [20] for the first equality)

$$\frac{\partial\ln H^2}{\partial\ln k} = \eta + \nu y^2 = \eta\left[1 - \Omega_{de}(t)\right], \tag{27}$$

where the second equality follows from our consistency condition.

In general the right-hand side evolves with time so an explicit dependence of k on H, such as $k \sim H^p$ which gives a constant left-hand side, without time explicitly entering, would be very special. Such a relation, which is sometimes assumed in the RG cosmology literature, will not in general be consistent.

A special case is when $\Omega_{de}(t) = \text{constant}$ is achieved through $\lambda = \text{constant}$ for all time, i.e. an exponential potential [24]. This situation implies that dark energy is either the only component if $\lambda^2 < 3$, or scales with the barotropic component otherwise; such a universe does not yield acceleration. Together with this must go that η and ν are constant. Thus, the assumption of the RG cutoff scale k being proportional to the Hubble scale, or some power of it, is extremely restricting, and does not lead to viable solutions describing our universe.

If we want to know how k asymptotically depends on H at the cosmological fixed point, we see that it can there have a power law relation with H. For example, in the dark energy potential dominated solution one gets asymptotically $k \sim H^{2/(\eta+\nu)}$. If one wanted the IR fixed point to return to the asymptotically safe UV fixed point of $\eta = -2$, $\nu = 4$, this would give $k \sim H$ in the future limit (but not for the present or all times in general).

In addition, note that astrophysical conditions exist on the flow of the gravitational coupling. Observations of the cosmic microwave background [25] and primordial nucleosynthesis abun-

Table 1
Cosmological attractor solutions under the renormalization group flow, including the values of the dark energy equation of state w, energy density Ω_{de}, and type of solution. When a variable is repeated under its column heading that means its value is moot.

x^2	y^2	λ	$d\ln k/dN$	w	Ω_{de}	Type
$\frac{\lambda^2}{6}$	$1-\frac{\lambda^2}{6}$	λ	0	$-1+\frac{\lambda^2}{3}$	1	Accelerating DE dominated
$\frac{3}{2}\frac{(1+w_b)^2}{\lambda^2}$	$\frac{3(1-w_b^2)}{2}\frac{1}{\lambda^2}$	λ	0	w_b	$\frac{3(1+w_b)}{\lambda^2}$	Scaling
$1+\frac{\eta}{\nu}$	$\frac{-\eta}{\nu}$	λ	$\frac{-6}{\nu}+\frac{\lambda}{\nu}\sqrt{\frac{6\nu}{\eta+\nu}}$	$1+\frac{2\eta}{\nu}$	1	Flowing DE dominated
0	0	λ	$d\ln k/dN$	w	0	Barotropic dominated
0	1	0	0	-1	1	DE potential dominated
1	0	λ	0	1	1	DE kinetic dominated
0	1	0	$d\ln k/dN$ but $\eta = 0 = \nu$	-1	1	no RG, de Sitter

dances [26] indicate that the coupling is constant to a precision of $\sim 10\%$ over a time comparable to the age of the universe. Thus,

$$\frac{\dot{G}}{G} = \frac{1}{H}\frac{d\ln G}{dN} = \frac{1}{H}\frac{d\ln k}{dN}\eta < \frac{0.1}{H}, \tag{28}$$

places a condition $\eta(d\ln k/dN) < 0.1$. This can be achieved either through a small magnitude of η or a slow flow $d\ln k/dN$ since primordial nucleosynthesis (~ 1 MeV scale). The dashed curve in Fig. 1 shows the condition on λ needed to give $d\ln k/dN = 0$ for the flowing DE critical point, for example. This will simultaneously also ensure that the matter equation of state $w_m = 0$.

5. Conclusions

In this Letter we have explored the quantum modifications to cosmological evolution at late times. For Einstein gravity and barotropic fluid and scalar field components, we considered the RG running of the gravitational coupling (Newton's constant) and the scalar field potential. Keeping the form of the equations of motion invariant under the RG evolution leads to a necessary consistency condition between the RG flow parameters and the cosmological quantities. This condition implies that one cannot adopt an arbitrary a priori relation between the RG cutoff scale k and the cosmological Hubble parameter $H(t)$.

From the RG influenced cosmological evolution equations, we have identified three classes of cosmological fixed points depending on the RG parameters: a dark energy dominated universe, a barotropic dominated universe, and solutions where the gravitational and potential couplings cease to flow. One can obtain an asymptotically de Sitter universe with $w = -1$ for specific choices of parameters, even if the potential has no intrinsic cosmological constant.

In general, due to the flow of the gravitational coupling the matter equation of state is not zero. This will affect structure formation, which is beyond the scope of this article, but the requirements on the parameters are similar to those directly on variation of G. We have considered cosmological constraints on $\dot{G}/G$ from cosmic microwave background and primordial nucleosynthesis observations and given the conditions necessary on the flow behavior. One can also satisfy both the matter equation of state and varying gravity requirements through specific choices of potential.

In this Letter we have not specified an explicit form of the scalar field potential. It would be interesting in future work to solve the RG equation explicitly for various specific potentials, such as those just mentioned, and see how the cosmological evolution develops toward the fixed points we have found. One could also consider the higher order terms beyond the conventional truncation as used here and see how the consistency condition is modified. This would serve as a test of the renormalization group formalism as usually applied to cosmology.

Acknowledgements

This work has been supported by the World Class University grant R32-2009-000-10130-0 of the National Research Foundation, Ministry of Education, Science and Technology of Korea. E.L. is also supported in part by the Director, Office of Science, Office of High Energy Physics of the U.S. Department of Energy under Contract No. DE-AC02-05CH11231.

References

[1] K.G. Wilson, J. Kogut, Phys. Repts. 12C (1974) 75.
[2] S. Weinberg, Phys. Rev. D 81 (2010) 083535.
[3] M. Niedermaier, M. Reuter, Liv. Rev. Relativity 9 (2006) 5.
[4] C. Wetterich, Phys. Lett. B 301 (1993) 90.
[5] M. Reuter, Phys. Rev. D 57 (1998) 971.
[6] A. Bonanno, M. Reuter, Phys. Rev. D 65 (2002) 043508.
[7] M. Reuter, F. Saueressig, JCAP 0509 (2005) 012.
[8] L. Xu, Mod. Phys. Lett. A 25 (2010) 377.
[9] J. Grande, J. Sola, H. Stefancic, Phys. Lett. B 645 (2007) 236.
[10] A. Babic, B. Guberina, R. Horvat, H. Stefancic, Phys. Rev. D 71 (2005) 124041.
[11] M. Reuter, H. Weyer, Phys. Rev. D 69 (2004) 104022.
[12] F. Perrotta, C. Baccigalupi, S. Matarrese, Phys. Rev. D 61 (2000) 023507.
[13] N. Bartolo, M. Pietroni, Phys. Rev. D 61 (2000) 023518.
[14] C. Baccigalupi, S. Matarrese, F. Perrotta, Phys. Rev. D 62 (2000) 123510.
[15] T. Chiba, Phys. Rev. D 64 (2001) 103503.
[16] V. Faraoni, M.N. Jensen, Class. Quant. Grav. 23 (2006) 3005.
[17] P. Minkowski, Phys. Lett. B 71 (1977) 419.
[18] A. Zee, Phys. Rev. Lett. 42 (1979) 417.
[19] S.L. Adler, Rev. Mod. Phys. 54 (1982) 729.
[20] M. Hindmarsh, D. Litim, C. Rahmede, JCAP 1107 (2011) 019.
[21] B. Koch, I. Ramirez, Class. Quant. Grav. 28 (2011) 055008.
[22] E.J. Copeland, A.R. Liddle, D. Wands, Phys. Rev. D 57 (1998) 4686.
[23] B. Ratra, P.J.E. Peebles, Phys. Rev. D 37 (1988) 3406.
[24] C. Wetterich, Nucl. Phys. B 302 (1988) 668.
[25] S. Galli, A. Melchiorri, G.F. Smoot, O. Zahn, Phys. Rev. D 80 (2009) 023508.
[26] J.-P. Uzan, Liv. Rev. Relativity 14 (2011) 2.