Journal Club 2014

노트 제목 2014-03-21

J. Cardy : Therm. & Revivals after Quantum Quench in CFT
arXiv 1403.3040

(Quench: suppress dyn. fermions)

H : time evolution of $|\Psi_0\rangle$ (not eigenstate), pure

can reach a stationary state ?

be described by a thermal density matrix ?

Can be addressed in cases of exactly solvable or AdS/CFT (BH)

[1] Calabrese Cardy 2006 (infinite-size system)

H for 1+1 CFT with a particular $|\Psi_0\rangle$

- correlation within a subsystem of length l becomes stationary after $t \approx \frac{l}{2}$
- becomes thermal at $T \approx$ Energy density
- entanglement entropy $S_l \sim$ Gibbs entropy at T.

L-mover , R-mover quasi particles entangled over a length scale $\sim \beta$

In the thermodynamic limit $L \to \infty$

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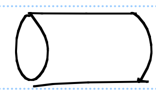
finite L thermalization : overlap between reduced density matrix and thermal mixed state

~ 1 $\quad t > l/2$ as $(t - l/2)\beta \to \infty$

until $t < \frac{L-l}{2}$

fidelity $\quad F(t) = |\langle \Psi_0 | e^{-iHt} | \Psi_0 \rangle| \Longleftrightarrow$



has singular points when $\frac{t}{L} = \frac{m}{n}$

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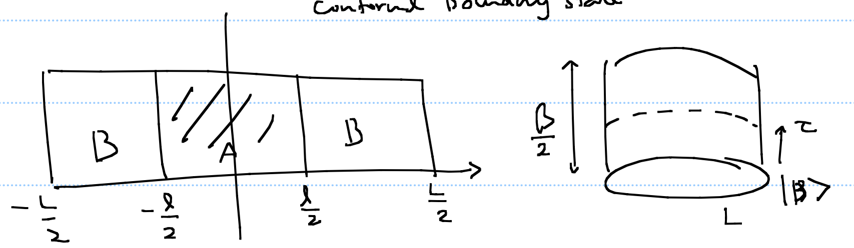
let $|\Psi_0\rangle$: ground-state of a pert. H ; $\quad H_{\text{CFT}} + \lambda \int \Phi \, dx$

Φ: relevant , massive m

let $mL \gg 1$

$\hookrightarrow$ let $|\Psi_0\rangle \propto e^{-\frac{\beta}{4}H} |B\rangle$

$|B\rangle$: Conformal Boundary state



$$\rho_A(\beta, \tau) = \mathrm{Tr}_{\mathcal{H}_B}\left(e^{-\tau H} e^{-\frac{1}{4}\beta H} |B\rangle \right)\left(\langle B| e^{-\frac{\beta}{4}H} e^{\tau H} \right)$$

$t=0$ $\boxed{|\psi_0\rangle}$ $\quad |\psi_0\rangle$ ground state of H_0

$t>0$ time evolution by H ← some parameter

"quench" : variation

$\boxed{|\psi(t)\rangle} = e^{-iHt}|\psi_0\rangle$

$|\psi_0\rangle$ quantum recurrence for finite d.o.f.

"Thermodynamic limit : infi d.o.f"

need not come back.

Then $|\psi(t)\rangle \sim$ stationary is possible as $t \gg 1$?

(or correlation functions $\langle \mathcal{O}(\vec{r},t)\,\mathcal{O}(0,0)\rangle$)

WHY?

* equilibration of quantum system

* quantum coherence is not maintained by decoherence effect for relatively long time

(Experimental difficulty) ⟼ now cold atom system is available!

Feshbach resonance tunes coupling to any value

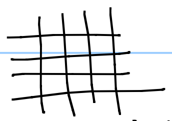
Coherent nonequilibrium has been measured (T. Kinoshita et al)

Goal

general features of quantum quench.

some results

- $t\to\infty$ $|\psi(t)\rangle$: Gibbs ensemble
- quantum in d $\equiv$ classi. in $d+1$



$|\psi_0\rangle$ ← not eigenstate of H

d-dim lattice + time

Hamiltonian H

$$\langle \mathcal{O}(t,\{\vec{r}_i\})\rangle \equiv \langle\psi_0|e^{iHt}\,\mathcal{O}(\{\vec{r}_i\})e^{-iHt}|\psi_0\rangle$$

$$\langle \mathcal{O}(t,\{\vec{r}_i\})\rangle_\epsilon \equiv \frac{\langle\psi_0|e^{iHt-\epsilon H}\,\mathcal{O}(\{\vec{r}_i\})e^{-iHt-\epsilon H}|\psi_0\rangle}{\langle\psi_0|e^{-2\epsilon H}|\psi_0\rangle}$$ will send $\epsilon\to 0$

$\boxed{\tau_{1,2} = \mp\epsilon + \hat{\tau}}$

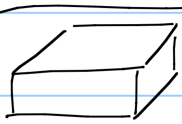
fixed

$t = i\hat{\tau}$

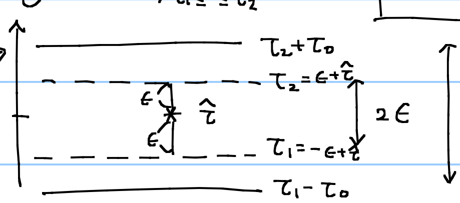
Path integral $= \underbrace{\int [\mathcal{D}\phi(\tau,\vec{r})]\,\langle\psi_0|\phi(\tau_2,\vec{r})\rangle\langle\phi(\tau_1,\vec{r})|\psi_0\rangle\ \mathcal{O}(\{\vec{r}_i\})\,e^{-\int_{\tau_1}^{\tau_2} L[\phi]d\tau}}$ with $\tau_1,\tau_2 = \pm\epsilon - it$

(d+1) dim

↳ $\tau_1 \le \tau \le \tau_2$



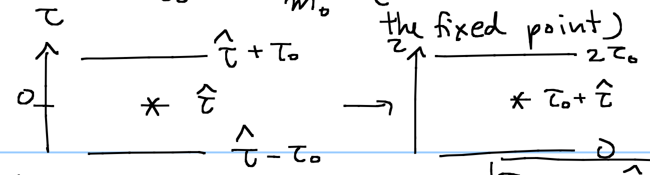
RG fixed point



$\tau_{1,2} = \hat{\tau} \pm \epsilon$

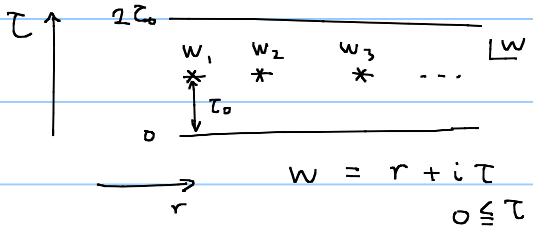
(Conformal Boundary state) $\quad \tau_0 \sim \frac{1}{m_0}$ (deviation from the fixed point)

Now take $\epsilon \to 0$



$$\boxed{\tau = \tau_0 + \hat{\tau}}$$

Dynamical correlator : $t_1, t_2, \cdots$



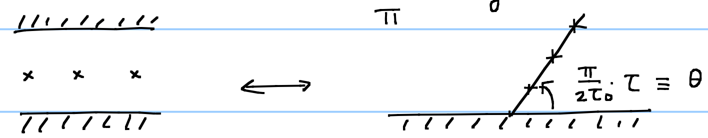
$(\mathrm{Im}\, w = \tau)$

$$w = r + i\tau$$

$$0 \leq \tau \leq 2\tau_0$$

Conformal map

$$w = \frac{2\tau_0}{\pi} \log z \quad \text{or} \quad z = e^{\frac{\pi}{2\tau_0} w}$$



$$w' = \frac{2\tau_0}{\pi z}$$

$$\left\langle \prod_i \Phi_i(w_i) \right\rangle_{\text{strip}} = \prod_i |w'(z_i)|^{-x_i} \underbrace{\left\langle \prod_i \Phi_i(z_i) \right\rangle_{\text{UHP}}}_{\text{a lot easier}}$$

$$\boxed{w' = \frac{2\tau_0}{\pi} e^{-\frac{\pi}{2\tau_0} w}}$$

(Ex) one-point

$$\langle \Phi(z) \rangle_{\text{UHP}} = \frac{A_b^{\Phi}}{|z-\bar{z}|^{x}}$$

$$\to \quad \langle \Phi(w) \rangle_{st} = \frac{A_b^{\Phi} |w'(z)|^{-x}}{\left| e^{\frac{\pi}{2\tau_0} w} - e^{\frac{\pi}{2\tau_0}\bar{w}} \right|^{x}} = \frac{A_b^{\Phi} \left(\frac{2\tau_0}{\pi}\right)^{-x} \left| \cancel{e^{\frac{\pi x}{2\tau_0}(r+i\tau)}} \right|}{\cancel{e^{\frac{\pi r}{2\tau_0} x}} \left| e^{\frac{i\pi}{2\tau_0}\tau} - e^{-\frac{i\pi}{2\tau_0}\tau} \right|^{x}}$$

$$= A_b^{\Phi} \left[\frac{\pi}{4\tau_0} \frac{1}{\sin\left(\frac{\pi\tau}{2\tau_0}\right)} \right]^{x}$$

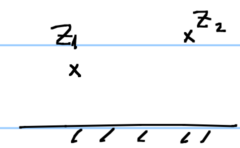
$$\sin \frac{\pi\tau}{2\tau_0} = \sin \frac{\pi}{2}\left(1 - \frac{it}{\tau_0}\right) \quad \leftarrow \quad \tau = \tau_0 + \hat{\tau} = \tau_0 - it$$

$$= \cosh \frac{\pi t}{2\tau_0}$$

$$\therefore \quad \langle \Phi \rangle = A_b^{\Phi} \left(\frac{\pi}{4\tau_0} \frac{1}{\mathrm{ch} \frac{\pi t}{2\tau_0}} \right)^{x} \underset{t \gg \tau_0}{\sim} A_b^{\Phi} \left(\frac{\pi}{2\tau_0}\right)^{x} e^{-\frac{x\pi t}{2\tau_0}} \quad //$$

$\boxed{\text{2-pt correlation}}$

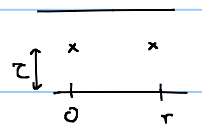
a free boson on UHP



$$\langle \Phi(z_1) \Phi(z_2) \rangle_{\text{UHP}} = \left(\frac{z_{1\bar{2}}\, z_{2\bar{1}}}{z_{12}\, z_{\bar{1}\bar{2}}\, z_{1\bar{1}}\, z_{2\bar{2}}} \right)^{x}$$

$\downarrow$

strip



$$\langle \Phi(r,\tau) \Phi(0,\tau) \rangle_{st} = |w'(z_1)|^{-x} |w'(z_2)|^{-x} \langle \Phi(z_1) \Phi(z_2) \rangle_{\text{UHP}}$$

$$= \left(\frac{2\tau_0}{\pi}\right)^{-2x} e^{+\frac{\pi}{2\tau_0}(w_1 + w_2) x} \left[\quad \wr \quad \right]$$

$$= \left[\left(\frac{\pi}{2\tau_0}\right)^2 \frac{\mathrm{ch}\left(\frac{\pi r}{2\tau_0}\right) - \cos\left(\frac{\pi\tau}{\tau_0}\right)}{8\, \mathrm{sh}^2 \frac{\pi r}{4\tau_0}\, \sin^2 \frac{\pi\tau}{2\tau_0}} \right]^{x}$$

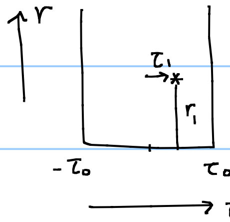
take $\tau = \tau_0 + it$

$$= \left[\left(\frac{\pi}{2\tau_0}\right)^2 \frac{ch(\frac{\pi r}{2\tau_0}) + \cosh(\frac{\pi t}{\tau_0})}{8\, sh^2 \frac{\pi r}{4\tau_0}\, ch^2 \frac{\pi t}{2\tau_0}} \right]^x$$

$$\sim \begin{cases} e^{-\frac{x\pi t}{\tau_0}} & \frac{r}{2} \gg t \gg t_0 \\ e^{-\frac{x\pi r}{2\tau_0}} & t \gg \frac{r}{2} \gg t_0 \end{cases}$$

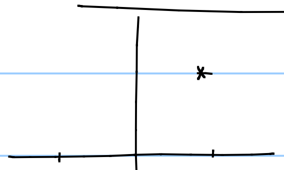
$t = t^* = \frac{r}{2}$ transition

Evolution with boundaries



$w = \tau + ir \to z = \sin \frac{\pi w}{2\tau_0} = \sin\left(\frac{\pi\tau}{2\tau_0} + i\frac{\pi r}{2\tau_0}\right)$

$= \sin\tau\, ch\, r + i \cos\tau\, sh\, r$



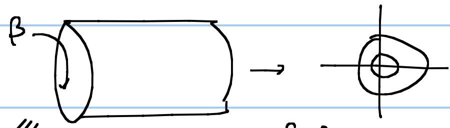
$$\langle \Phi \rangle_{1st} \propto \left[\frac{ch\frac{\pi t}{\tau_0} + ch\frac{\pi r}{\tau_0}}{ch^2\frac{\pi t}{2\tau_0}\, sh^2\frac{\pi r}{2\tau_0}} \right]^{x/2} \sim \begin{cases} e^{-\frac{\pi x t}{2\tau_0}} & t < r \\ e^{-\frac{\pi x r}{2\tau_0}} & t > r \end{cases}$$

Universal behaviour

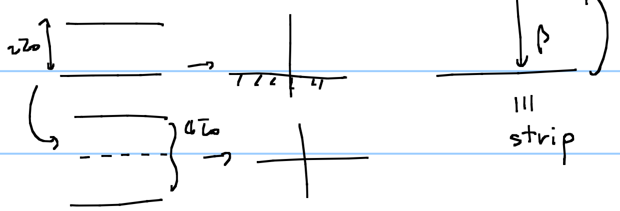
transition when $t = \frac{r}{2} \equiv t^*$

$t \gg 1 \to e^{-\frac{x\pi r}{2\tau_0}}$

(cf) finite temperature correlation ;



$w = \frac{\beta}{2\pi} \log z$



$\to \beta = 4\tau_0$