B-type anomaly coefficients of holographic defects

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Interfaces and defects in strongly coupled matter research group

APCTP Focus Program "Integrability duality and related topics"

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based on my work with M. de Leeuw, C. Kristjansen and M. Volk, PLB **846** (2023) 138235 [arxiv:2307.10946], as well as work in progress

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Section 1

Introduction

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- Weak/strong coupling dilemma: gauge and the string theory couplings are inversely proportional... the two perturbative regimes are disconnected from each other... testing AdS/CFT is practically impossible!

Integrability!

- Nonetheless, there still exists a large number of nontrivial tests from weak ($\lambda \to 0$) to strong 't Hooft coupling ($\lambda \to \infty$) which confirms the validity of the AdS/CFT correspondence for large values of N_c.

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- According to the *dictionary* of the AdS/CFT duality, the above operators of $\mathcal{N} = 4$, SYM are dual to type IIB string theory states in $AdS_5 \times S^5$...

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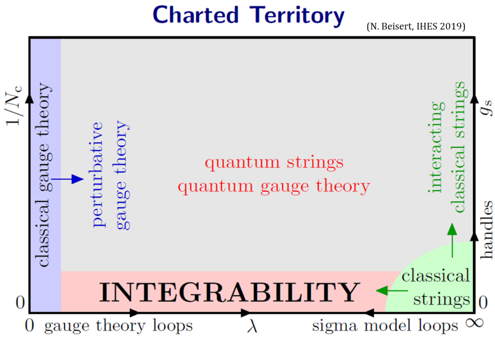

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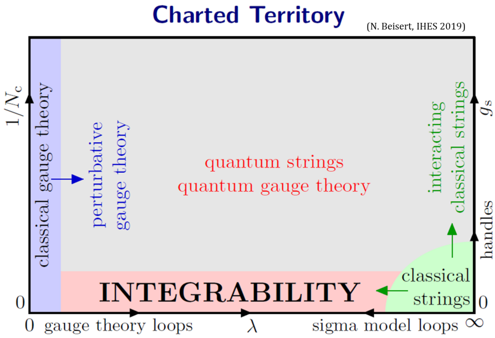

- Ideally, we would like to solve the theory… not only its spectrum… where by *solve* we mean the calculation of the theory's observables: spectrum, correlation functions, scattering amplitudes, Wilson loop expectation values, etc…

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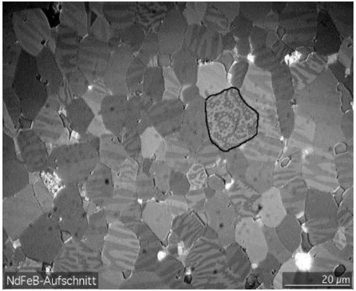

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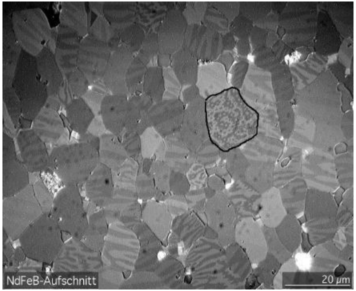

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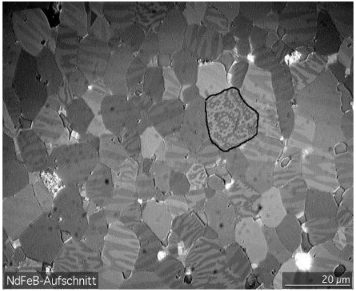

- The real-world gauge theories we would like to study at strong coupling (such as QCD) are neither finite, nor supersymmetric, nor integrable, (or holographic?)... In other words, we need less symmetry!

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- Let us first see how AdS/dCFT is obtained from AdS/CFT...

The AdS/CFT correspondence

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On the lhs, $\mathcal{N}=4$, super Yang-Mills (SYM) theory is a 4-dimensional superconformal gauge theory:

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- Spectral problem solved (Gromov-Kazakov-Leurent-Volin, 2013)... solution of full planar theory by computing all observables (correlators, scattering amplitudes, Wilson loops, etc) underway...
- Half-BPS boundary conditions in $\mathcal{N}=4$ SYM were studied by Gaiotto-Witten (2008)...

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The $\mathrm{AdS}_5/\mathrm{CFT}_4$ correspondence states that:

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Maldacena (1997)

Type IIB superstring theory on $\mathrm{AdS}_5 \times \mathrm{S}^5$ is described by a nonlinear σ-model on a supercoset:

$$\mathrm{AdS}_5 \times \mathrm{S}^5 = \frac{SO(4,2)}{SO(4,1)} \times \frac{SO(6)}{SO(5)} \subseteq \frac{PSU(2,2|4)}{SO(4,1) \times SO(5)}.$$

Green-Schwarz superstring action on $\mathrm{AdS}_5 \times \mathrm{S}^5$ is a WZW sigma model (Metsaev-Tseytlin, 1998):

$$S = -\frac{T_2}{2} \int \ell^2 \mathrm{str}\left[J^{(2)} \wedge \star J^{(2)} + J^{(1)} \wedge J^{(3)}\right], \qquad J \equiv \mathfrak{g}^{-1} d\mathfrak{g}, \qquad T_2 \equiv \frac{1}{2\pi\alpha'} = \frac{\sqrt{\lambda}}{2\pi\ell^2}.$$

The $\mathrm{AdS}_5 \times \mathrm{S}^5$ supercoset is a semi-symmetric space, i.e. its elements afford a $\mathbb{Z}_4$ decomposition:

$$J = J^{(0)} + J^{(1)} + J^{(2)} + J^{(3)}, \qquad \Omega\left[J^{(n)}\right] = i^n J^{(n)}, \qquad \Omega\left(M\right) = -\mathcal{K} M^{\mathrm{st}} \mathcal{K}^{-1}, \quad \mathcal{K} = \begin{bmatrix} \gamma_{13} & 0 \\ 0 & \gamma_{13} \end{bmatrix}.$$

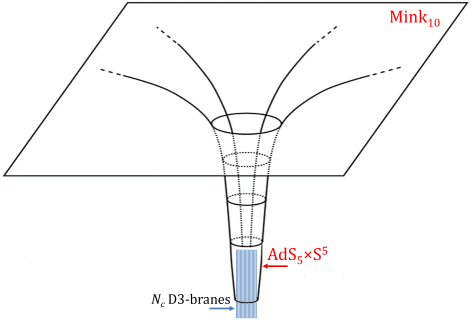
Nonlinear sigma models on semi-symmetric spaces are classically integrable (Bena-Polchinski-Roiban, 2003)...

Section 2

Probe-brane defect systems

The D3-D5 system: bulk geometry

Type IIB string theory on $AdS_5 \times S^5$ is encountered very close to a system of N_c coincident D3-branes:

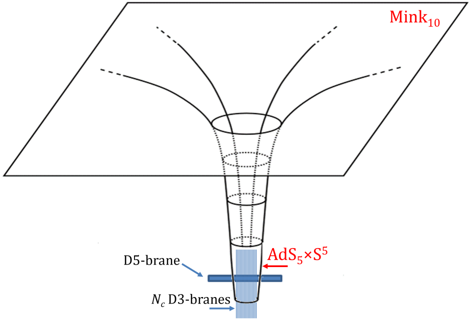


The D3-branes extend along x_1, x_2, x_3...

	t	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9
D3	•	•	•	•						

The D3-D5 system: bulk geometry

Type IIB string theory on $AdS_5 \times S^5$ is encountered very close to a system of N_c coincident D3-branes:



Now insert a single (probe) D5-brane at $x_3 = x_7 = x_8 = x_9 = 0$...

	t	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9
D3	•	•	•	•						
D5	•	•	•		•	•	•			

The D3-D5 system: bulk geometry

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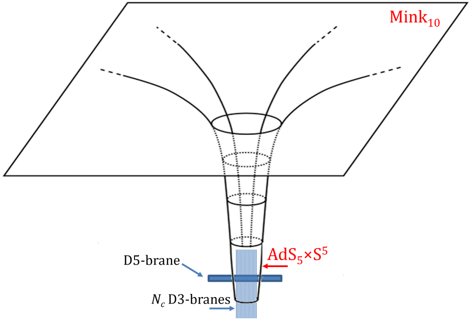
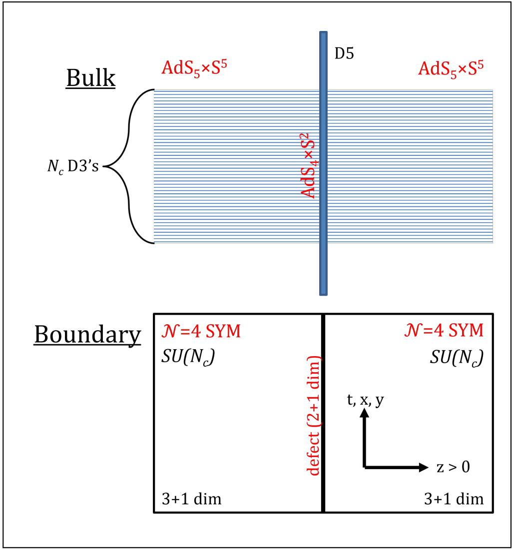


Now insert a single (probe) D5-brane at $x_3 = x_7 = x_8 = x_9 = 0$...

	t	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9
D3	•	•	•	•						
D5	•	•	•		•	•	•			

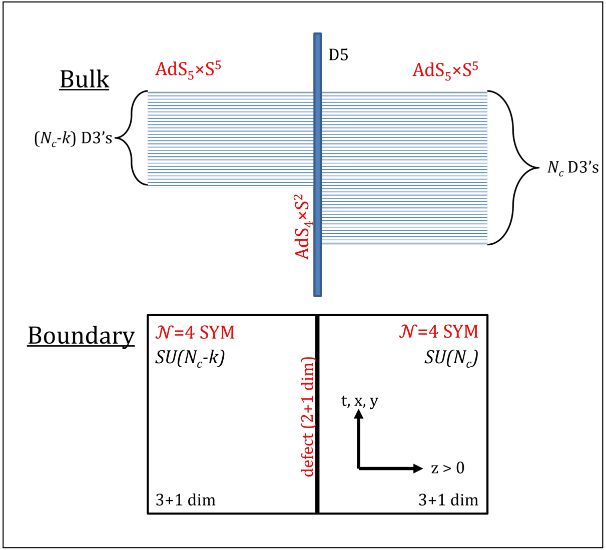
... its geometry will be $AdS_4 \times S^2$ (Karch-Randall, 2001b)...

The D3-D5 system: description



- The defect reduces the total bosonic symmetry of the system from $SO(4,2) \times SO(6)$ to $SO(3,2) \times SO(3) \times SO(3)$. The corresponding superalgebra $\mathfrak{psu}(2,2|4)$ becomes $\mathfrak{osp}(4|4)$. Supersymmetry studied by Domokos-Royston (2022)...
- The D3-D5 system describes IIB string theory on $\text{AdS}_5 \times \text{S}^5$ bisected by a D5 brane with worldvolume geometry $\text{AdS}_4 \times \text{S}^2$.
- The D5-brane is stable... the tachyonic instability in the fluctuations of ψ does not violate the BF bound (Karch-Randall, 2001b)...
- The probe D5-brane is classically integrable... i.e. infinite conserved charges for open strings with D5-brane BCs (Dekel-Oz, 2011)...
- The dual field theory is still $SU(N_c)$, $\mathcal{N} = 4$ SYM in $3+1$ dimensions, that interacts with a CFT living on the $2+1$ dimensional defect: $S = S_{\mathcal{N}=4} + S_{2+1}$ (DeWolfe-Freedman-Ooguri, 2001).
- $\mathcal{N} = 4$ spin chain not modified by the presence of the defect... open spin chain ending on defect fields remains integrable (DeWolfe-Mann, 2004)...

The $(D3\text{-}D5)_k$ dSCFT



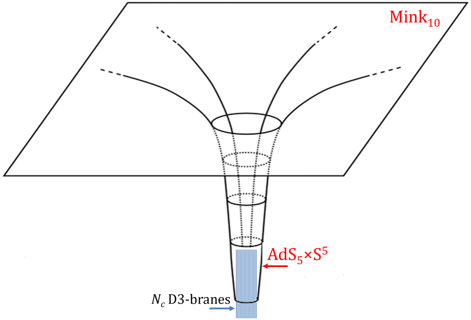
- Despite stability, add $k \neq 0$ units of background magnetic flux over S^2… brane geometry $AdS_4 \times S^2$…
- D5-brane with flux preserves classical integrability of open strings (Zarembo-GL, 2021)…
- The SCFT gauge group $SU(N_c) \times SU(N_c)$ breaks to $SU(N_c - k) \times SU(N_c)$…
- Equivalently, the fields of $\mathcal{N} = 4$ SYM develop nonzero vevs (Karch-Randall, 2001b)… dCFT correlators = Higgs condensates of gauge-invariant operators of $\mathcal{N} = 4$ SYM (Nagasaki-Yamaguchi, 2012)…
- Matrix product states… overlaps with Bethe states… Scalar one-point functions (de Leeuw, Kristjansen, Zarembo, 2015)… closed-form det formulas… integrable quench criteria satisfied (Piroli, Pozsgay, Vernier, 2017; de Leeuw-Kristjansen-GL, 2018)…
- Two-point functions of (spin-2) stress tensor, displacement operator, anomaly coefficients (de Leeuw-Kristjansen-GL-Volk 2023)… More below!
- Strong-coupling computations were recently set up (Georgiou-GL-Zoakos, 2023)…

Subsection 2

The D3-D7 probe-brane system

The D3-D7 system: bulk geometry

IIB string theory on $AdS_5 \times S^5$ is encountered very close to a system of N_c coincident D3-branes:

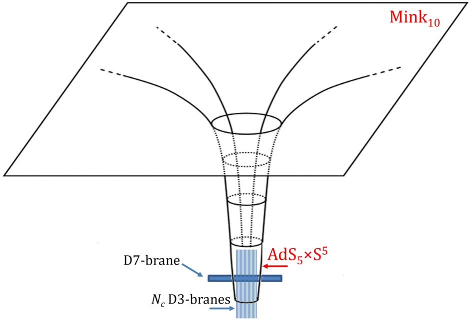


The D3-branes extend along x_1, x_2, x_3...

	t	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9
D3	●	●	●	●						

The D3-D7 system: bulk geometry

IIB string theory on $AdS_5 \times S^5$ is encountered very close to a system of N_c coincident D3-branes:

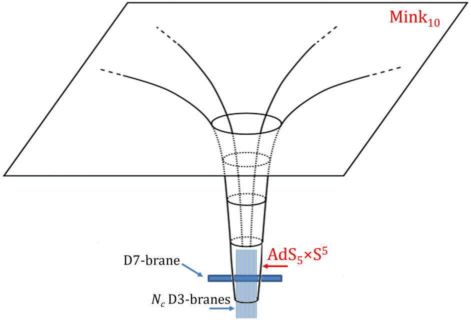


Now insert a single D7-brane at $x_3 = x_9 = 0$...

	t	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9
D3	•	•	•	•						
D7	•	•	•		•	•	•	•	•	

The D3-D7 system: bulk geometry

IIB string theory on $AdS_5 \times S^5$ is encountered very close to a system of N_c coincident D3-branes:

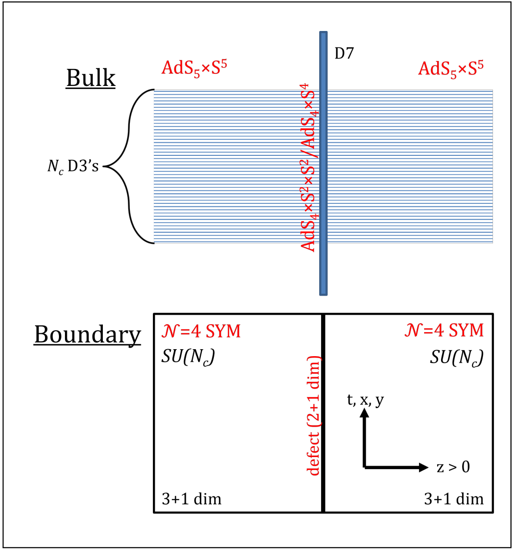


Now insert a single D7-brane at $x_3 = x_9 = 0$... its geometry will be either $AdS_4 \times S^4$ or $AdS_4 \times S^2 \times S^2$...

	t	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9
D3	●	●	●	●						
D7	●	●	●		●	●	●	●	●	

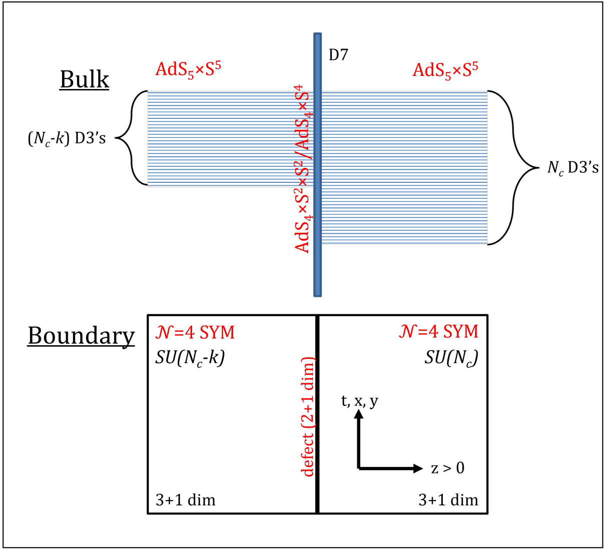
(Davis-Kraus-Shah, 2008; Myers-Wapler, 2008; Bergman-Jokela-Lifschytz-Lippert, 2010)...

The D3-D7 system: description



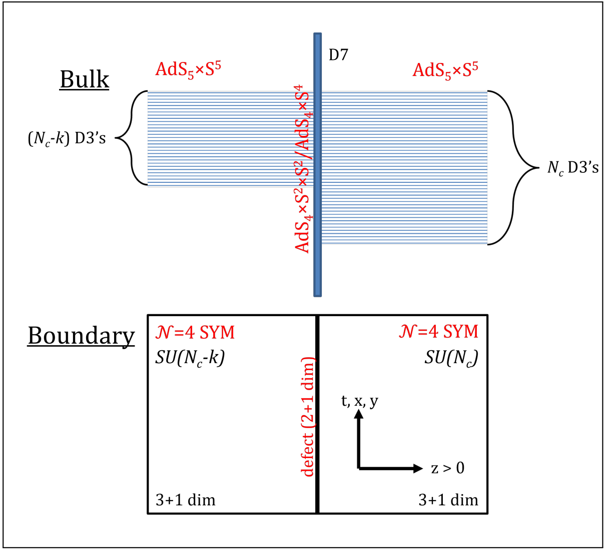
- The defect reduces the total bosonic symmetry of the system from $SO(4,2) \times SO(6)$ to either $SO(3,2) \times SO(5)$ or $SO(3,2) \times SO(3) \times SO(3)$... All susy broken! (relative brane codimension in flat space: $\#_{\text{ND}} = 6 \to$ no unbroken susy)...
- The D3-D7 system describes IIB string theory on $\text{AdS}_5 \times \text{S}^5$ bisected by a D7-brane with worldvolume geometry $\text{AdS}_4 \times \text{S}^4$ or $\text{S}^2 \times \text{S}^2$... maximal S^4 & $\text{S}^2 \times \text{S}^2$ sit on the equator of S^5...
- The D7-branes are unstable: tachyonic instabilities in fluctuations violate the BF bound (Davis-Kraus-Shah, 2008; Bergman-Jokela-Lifschytz-Lippert, 2010)... S^4 and $\text{S}^2 \times \text{S}^2$ "slip-off" (either side of) the S^5 equator, collapsing to points...
- Various ways to lift the instability... embed D7 in full D3-brane geometry instead of near-horizon (Davis-Kraus-Shah, 2008)... impose an AdS cutoff Λ (Kutasov-Lin-Parnachev, 2011; Mezzalira-Parnachev, 2015)... add instanton flux on S^4 (Myers-Wapler, 2008), and magnetic flux on $\text{S}^2 \times \text{S}^2$ (Bergman-Jokela-Lifschytz-Lippert, 2010)...
- The dual field theory is still $SU(N_c)$, $\mathcal{N} = 4$ SYM in $3+1$ dimensions, that interacts with a CFT living on the $2+1$ dimensional defect: $S = S_{\mathcal{N}=4} + S_{2+1}$... boundary degrees of freedom are fermions (Rey, 2009)...

The $(\text{D3-D7})_k$ system



- To stabilize the D7-brane, we add a (non-abelian) instanton bundle through its S^4 component (Myers-Wapler, 2008) and an (abelian) magnetic flux through each S^2 (Bergman-Jokela-Lifschytz-Lippert, 2010)...
- This forces exactly k (flux units) of the N_c D3-branes ($N_c \gg k$) to end on the D7-brane...
- The homogeneous instanton flux is non-abelian... study of classical string integrability hard in the $SO(5)$ symmetric case... the $SU(2) \times SU(2)$ symmetric system is most probably not integrable...
- On the gauge theory side, gauge group $SU(N_c) \times SU(N_c)$ breaks to $SU(N_c) \times SU(N_c - k)$...
- Equivalently, the fields of $\mathcal{N} = 4$ SYM develop nonzero vevs... dCFT correlators = Higgs condensates of gauge-invariant operators of $\mathcal{N} = 4$ SYM...
- Matrix product states... overlaps with Bethe states... scalar one-point functions (de Leeuw-Kristjansen-GL, 2016)... integrable quench criteria satisfied in the $SO(5)$ symmetric case (Piroli, Pozsgay, Vernier, 2017; de Leeuw-Kristjansen-GL, 2018)...

The (D3-D7)$_k$ system

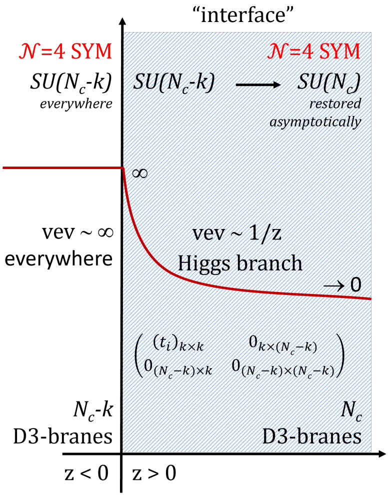


- Yet another sign of integrability of the $SO(5)$ symmetric system are closed-form determinant formulas which have been found for all scalar one-point functions (de Leeuw-Gombor-Kristjansen-GL-Pozsgay, 2019)...
- Weak-coupling analysis also provides evidence of non-integrability for the $SU(2) \times SU(2)$ symmetric system (de Leeuw-Kristjansen-Vardinghus, 2019)...
- Two-point functions of the (spin-2) stress tensor, displacement operator, anomalies... More below...
- Strong-coupling computations were recently set up (Georgiou-GL-Zoakos, 2023)...

Subsection 3

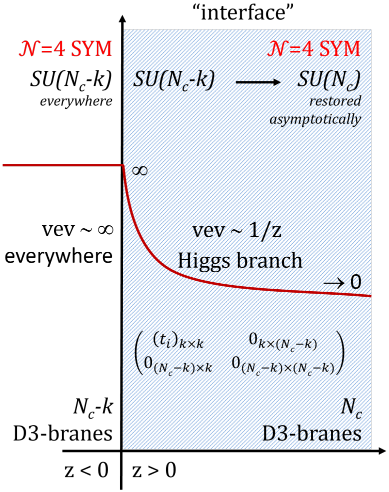
One-point functions

The D3-D5 interface: $SU(2) \times SU(2)$ symmetry



- An interface is a wall between two (different/same) QFTs...
- It can be described by means of classical solutions that are known as "fuzzy-funnel" solutions (Constable-Myers-Tafjord, 1999 & 2001)...

The D3-D5 interface: $SU(2) \times SU(2)$ symmetry



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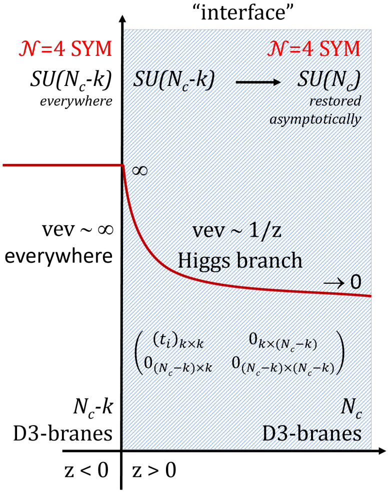
- Here, an interface (situated at $z = 0$) separates the $SU(N_c)$ and $SU(N_c - k)$ regions of the $(\text{D3-D5})_k$ dCFT...

The D3-D5 interface: $SU(2) \times SU(2)$ symmetry



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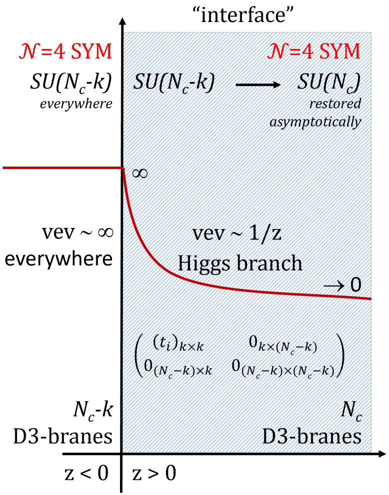
- Here, an interface (situated at $z = 0$) separates the $SU(N_c)$ and $SU(N_c - k)$ regions of the $(\text{D3-D5})_k$ dCFT...
- For no vectors/fermions, we want to solve the equations of motion for the scalar fields of $\mathcal{N} = 4$ SYM:

$$A_\mu = \psi_a = 0, \qquad \frac{d^2\varphi_i}{dz^2} = [\varphi_j, [\varphi_j, \varphi_i]], \quad i, j = 1, \ldots, 6.$$

The D3-D5 interface: $SU(2) \times SU(2)$ symmetry



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$$A_\mu = \psi_a = 0, \qquad \frac{d^2\varphi_i}{dz^2} = [\varphi_j, [\varphi_j, \varphi_i]], \qquad i, j = 1, \ldots, 6.$$

- A manifestly $SO(3) \simeq SU(2)$ symmetric solution is given by $(z > 0)$:

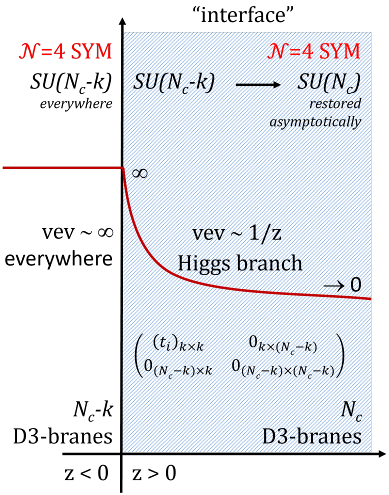
$$\varphi_{2i-1}(z) = \frac{1}{z}\begin{bmatrix} (t_i)_{k\times k} & 0_{k\times(N_c-k)} \\ 0_{(N_c-k)\times k} & 0_{(N_c-k)\times(N_c-k)} \end{bmatrix} \quad \& \quad \varphi_{2i} = 0,$$

Diaconescu (1996), Giveon-Kutasov (1998)

where the matrices t_i furnish a k-dimensional representation of $\mathfrak{su}(2)$:

$$[t_i, t_j] = i\epsilon_{ijk} t_k.$$

The D3-D5 interface: $SU(2) \times SU(2)$ symmetry



- An interface is a wall between two (different/same) QFTs...
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- Here, an interface (situated at $z = 0$) separates the $SU(N_c)$ and $SU(N_c - k)$ regions of the $(\text{D3-D5})_k$ dCFT...
- For no vectors/fermions, we want to solve the equations of motion for the scalar fields of $\mathcal{N} = 4$ SYM:

$$A_\mu = \psi_a = 0, \qquad \frac{d^2\varphi_i}{dz^2} = [\varphi_j, [\varphi_j, \varphi_i]], \quad i, j = 1, \ldots, 6.$$

- A manifestly $SO(3) \simeq SU(2)$ symmetric solution is given by $(z > 0)$:

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Diaconescu (1996), Giveon-Kutasov (1998)

- The solution also satisfies the Nahm equations:

$$\frac{d\varphi_i}{dz} = \frac{i}{2}\epsilon_{ijk}[\varphi_j, \varphi_k],$$

as expected for a half-BPS interface (Gaiotto-Witten, 2008)...

One-point functions

Following Nagasaki & Yamaguchi (2012), the one-point functions of local gauge-invariant scalar operators,

$$\langle \mathcal{O}(\mathrm{z}, \mathbf{x}) \rangle = \frac{\mathcal{C}}{\mathrm{z}^{\Delta}}, \qquad \mathrm{z} > 0,$$

can be calculated within the D3-D5 defect CFT from the corresponding fuzzy-funnel solution, for example:

$$\boxed{\mathcal{O}(z, \mathbf{x}) = \Psi^{\mu_1 \ldots \mu_L} \mathrm{tr}\left[\varphi_{2\mu_1 - 1} \ldots \varphi_{2\mu_L - 1}\right] \xrightarrow[\text{interface}]{SU(2)} \frac{1}{z^L} \cdot \Psi^{\mu_1 \ldots \mu_L} \mathrm{tr}\left[t_{\mu_1} \ldots t_{\mu_L}\right]}$$

where $\Psi^{\mu_1 \cdots \mu_L}$ is an $SO(6)$ symmetric tensor and the constant $\mathcal{C}$ is given by (MPS="*matrix product state*"),

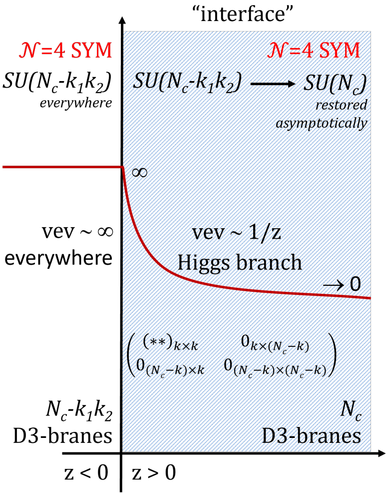
$$\mathcal{C} = \frac{1}{\sqrt{L}} \left(\frac{8\pi^2}{\lambda}\right)^{L/2} \cdot \frac{\langle \mathrm{MPS} | \Psi \rangle}{\langle \Psi | \Psi \rangle^{\frac{1}{2}}}, \qquad \left\{ \begin{array}{ll} \langle \mathrm{MPS} | \Psi \rangle \equiv \Psi^{\mu_1 \cdots \mu_L} \mathrm{tr}\left[t_{\mu_1} \ldots t_{\mu_L}\right] & (\text{"overlap"}) \\ \langle \Psi | \Psi \rangle \equiv \Psi^{\mu_1 \cdots \mu_L} \Psi_{\mu_1 \cdots \mu_L} & \end{array} \right\},$$

which ensures that the 2-point function will be normalized to unity ($\mathcal{O} \to (2\pi)^L \left(L\lambda^L\right)^{-1/2} \cdot \mathcal{O}$):

$$\langle \mathcal{O}(\mathrm{x}_1) \mathcal{O}(\mathrm{x}_2) \rangle = \frac{1}{|\mathrm{x}_1 - \mathrm{x}_2|^{2\Delta}},$$

within $SU(N_c)$, $\mathcal{N} = 4$ SYM (i.e. without the defect). Once more, we set $\mathrm{x}_i \equiv (\mathrm{z}_i, \mathbf{x}_i)$, where $\mathbf{x}_i \equiv \{\mathrm{x}_i^{(0,1,2)}\}$.

The D3-D7 interface: $SU(2) \times SU(2)$ symmetry



- To compute correlation functions in the dCFT that is dual to the $SU(2) \times SU(2)$ symmetric D3-D7 system, we set up the corresponding interface...
- The interface (placed at $z = 0$) separates the $SU(N_c)$ and $SU(N_c - k_1 k_2)$ regions of the $(\text{D3-D7})_{k_1 k_2}$ dCFT... It will be described by a fuzzy funnel solution...
- For no vectors/fermions, we want to solve the equations of motion for the scalar fields of $\mathcal{N} = 4$ SYM:

$$A_\mu = \psi_a = 0, \qquad \frac{d^2\varphi_i}{dz^2} = \left[\varphi_j, \left[\varphi_j, \varphi_i\right]\right], \quad i, j = 1, \ldots, 6.$$

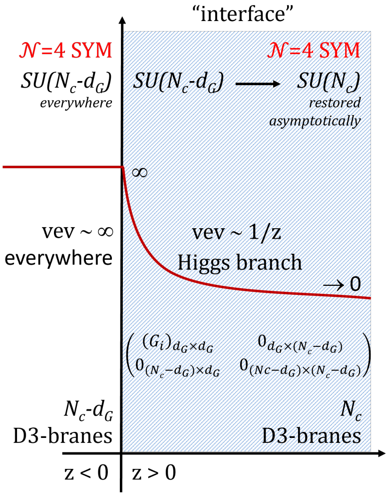
- The wanted $SU(2) \times SU(2) \subset SU(3,2) \times SU(2) \times SU(2)$ solution is:

$$\varphi_i(z) = -\frac{1}{z} \times \begin{cases} \left[(t_i)_{k_1} \otimes \mathbb{1}_{k_2}\right] \oplus 0_{(N_c - k_1 k_2)}, & i = 1, 2, 3 \\ \left[\mathbb{1}_{k_1} \otimes (t_i)_{k_2}\right] \oplus 0_{(N_c - k_1 k_2)}, & i = 4, 5, 6. \end{cases}$$

Kristjansen-Semenoff-Young (2012)

- The defect CFT is not supersymmetric so that the interface does not satisfy the Nahm equations...

The D3-D7 interface: $SO(5)$ symmetry



- The interface for the dCFT that is dual to the $SO(5)$ symmetric D3-D7 system (placed at $z = 0$) separates the $SU(N_c)$ and $SU(N_c - d_G)$ regions of the $(\text{D3-D7})_{d_G}$ dCFT... It will be described by a fuzzy funnel solution...
- For no vectors/fermions, we solve the equations of motion for the scalar fields of $\mathcal{N} = 4$ SYM:

$$A_\mu = \psi_a = 0, \qquad \frac{d^2\varphi_i}{dz^2} = \left[\varphi_j, \left[\varphi_j, \varphi_i\right]\right], \quad i, j = 1, \ldots, 6.$$

- A manifestly $SO(5) \subset SO(3,2) \times SO(5)$ symmetric solution is given by:

$$\varphi_i(z) = \frac{G_i \oplus 0_{(N_c-d_G)\times(N_c-d_G)}}{\sqrt{8}\,z}, \quad i = 1, \ldots, 5, \qquad \varphi_6 = 0\,.$$

Kristjansen-Semenoff-Young (2012)

- Once more, the defect CFT is not supersymmetric so that the interface does not satisfy the Nahm equations...
- The five $d_G \times d_G$ matrices G_i are known as the "fuzzy" S^4 matrices...

The fuzzy S^4 G-matrices

The five $d_G \times d_G$ fuzzy S^4 matrices (G-matrices) G_i are given by:

$$G_i \equiv \Big[\underbrace{\overbrace{\gamma_i \otimes \mathbb{1}_4 \otimes \ldots \otimes \mathbb{1}_4}^{n \text{ factors}} + \mathbb{1}_4 \otimes \gamma_i \otimes \ldots \otimes \mathbb{1}_4 + \ldots + \mathbb{1}_4 \otimes \ldots \otimes \mathbb{1}_4 \otimes \gamma_i}_{n \text{ terms}}\Big]_{\text{sym}} \qquad (i = 1, \ldots, 5),$$

Castelino-Lee-Taylor (1997)

where γ_i are the five 4×4 Euclidean Dirac matrices:

$$\gamma_i = \begin{pmatrix} 0 & -i\sigma_i \\ i\sigma_i & 0 \end{pmatrix}, \quad i = 1, 2, 3, \qquad \gamma_4 = \begin{pmatrix} 0 & \mathbb{1}_2 \\ \mathbb{1}_2 & 0 \end{pmatrix}, \qquad \gamma_5 = \begin{pmatrix} \mathbb{1}_2 & 0 \\ 0 & -\mathbb{1}_2 \end{pmatrix},$$

and σ_i are the three Pauli matrices. The ten commutators of the five G-matrices,

$$G_{ij} \equiv \frac{1}{2}\left[G_i, G_j\right],$$

furnish a d_G-dimensional (anti-hermitian) irreducible representation of $\mathfrak{so}(5) \simeq \mathfrak{sp}(4)$:

$$\left[G_{ij}, G_{kl}\right] = 2\left(\delta_{jk} G_{il} + \delta_{il} G_{jk} - \delta_{ik} G_{jl} - \delta_{jl} G_{ik}\right).$$

The fuzzy S^4 G-matrices

The dimension of the G-matrices is equal to the instanton number $d_G = (n+1)(n+2)(n+3)/6$:

n	1	2	3	4	5	6	7	8	9	10	...
d_G	4	10	20	35	56	84	120	165	220	286	...

E.g., for $n = 2$, here are the 10×10 G-matrices:

$$G_1 = \begin{pmatrix} 0 & 0 & 0 & -i\sqrt{2} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -i & 0 & 0 & 0 & -i & 0 & 0 & 0 \\ 0 & i & 0 & 0 & 0 & 0 & 0 & 0 & -i & 0 \\ i\sqrt{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -i\sqrt{2} \\ 0 & 0 & 0 & 0 & 0 & -i\sqrt{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & i\sqrt{2} & 0 & 0 & -i\sqrt{2} & 0 & 0 \\ 0 & i & 0 & 0 & 0 & 0 & 0 & 0 & -i & 0 \\ 0 & 0 & 0 & 0 & 0 & i\sqrt{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & i & 0 & 0 & 0 & i & 0 & 0 & 0 \\ 0 & 0 & 0 & i\sqrt{2} & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}, \quad G_2 = \begin{pmatrix} 0 & 0 & 0 & -\sqrt{2} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ -\sqrt{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\sqrt{2} \\ 0 & 0 & 0 & 0 & 0 & \sqrt{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \sqrt{2} & 0 & 0 & \sqrt{2} & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & \sqrt{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\sqrt{2} & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix},$$

$$G_3 = \begin{pmatrix} 0 & 0 & -i\sqrt{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & i & 0 & -i & 0 & 0 & 0 & 0 \\ i\sqrt{2} & 0 & 0 & 0 & 0 & 0 & 0 & -i\sqrt{2} & 0 & 0 \\ 0 & -i & 0 & 0 & 0 & 0 & 0 & 0 & -i & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & i\sqrt{2} & 0 & 0 & 0 \\ 0 & i & 0 & 0 & 0 & 0 & 0 & 0 & i & 0 \\ 0 & 0 & 0 & 0 & -i\sqrt{2} & 0 & 0 & 0 & 0 & i\sqrt{2} \\ 0 & 0 & i\sqrt{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & i & 0 & -i & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -i\sqrt{2} & 0 & 0 & 0 \end{pmatrix}, \quad G_4 = \begin{pmatrix} 0 & 0 & \sqrt{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ \sqrt{2} & 0 & 0 & 0 & 0 & 0 & 0 & \sqrt{2} & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \sqrt{2} & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & \sqrt{2} & 0 & 0 & 0 & 0 & \sqrt{2} \\ 0 & 0 & \sqrt{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \sqrt{2} & 0 & 0 & 0 \end{pmatrix}, \quad G_5 = \begin{pmatrix} 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -2 \end{pmatrix}$$

One-point functions

One-point functions of local gauge-invariant scalar operators,

$$\langle \mathcal{O}(\mathrm{z}, \mathbf{x}) \rangle = \frac{\mathcal{C}}{\mathrm{z}^{\Delta}}, \qquad \mathrm{z} > 0,$$

can again be calculated within the D3-D7 defect CFT from the corresponding fuzzy funnel solution...

$$\boxed{\mathcal{O}(\mathrm{z}, \mathbf{x}) = \Psi^{i_1 \dots i_L} \mathrm{tr}\left[\varphi_{i_1} \dots \varphi_{i_L}\right] \xrightarrow[\text{interface}]{SO(5),\ SO(3)\times SO(3)} \frac{1}{z^L} \cdot \Psi^{i_1 \dots i_L} \mathrm{tr}\left[\tau_{i_1} \dots \tau_{i_L}\right]},$$

where the matrices τ_i are defined in terms of the corresponding fuzzy funnel solution:

$$\tau_i = \begin{cases} \left.\begin{array}{ll} G_i/\sqrt{8}, & i = 1, \dots, 5 \\ 0, & i = 6 \end{array}\right\}, & SO(5) \text{ symmetric interface} \\ \left.\begin{array}{ll} \left[(t_i)_{k_1} \otimes \mathbb{1}_{k_2}\right] \oplus 0_{(N_c - k_1 k_2)}, & i = 1, 2, 3 \\ \left[\mathbb{1}_{k_1} \otimes (t_i)_{k_2}\right] \oplus 0_{(N_c - k_1 k_2)}, & i = 4, 5, 6 \end{array}\right\}, & SO(3) \times SO(3) \text{ symmetric interface.} \end{cases}$$

Again, $\Psi^{i_1 \dots i_L}$ is an $\mathfrak{so}(6)$-symmetric tensor and the constant $\mathcal{C}$ is given by (MPS="*matrix product state*"),

$$\mathcal{C} = \frac{1}{\sqrt{L}} \left(\frac{\pi^2}{\lambda}\right)^{L/2} \cdot \frac{\langle \mathrm{MPS} | \Psi \rangle}{\langle \Psi | \Psi \rangle^{\frac{1}{2}}}, \qquad \left\{ \begin{array}{ll} \langle \mathrm{MPS} | \Psi \rangle \equiv \Psi^{i_1 \dots i_L} \mathrm{tr}\left[G_{i_1} \dots G_{i_L}\right] & (\text{"overlap"}) \\ \langle \Psi | \Psi \rangle \equiv \Psi^{i_1 \dots i_L} \Psi_{i_1 \dots i_L} & \end{array} \right\}.$$

Section 3

Defect anomaly coefficients

Defect anomalies

Even dimensional CFTs (in curved spacetimes) are afflicted by conformal/Weyl anomalies: the trace of the energy-momentum/stress tensor acquires non-vanishing expectation value that is given by (scheme-independent terms only)...

$$\langle T^{\mu}_{\mu} \rangle^{d=2n} = \frac{4}{d!\,\mathrm{Vol}[S^d]} \times \left[\sum_i c_i\, I_i - (-1)^{d/2} a_d\, E_d \right], \quad n = 1, 2, \ldots$$

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Odd dimensional (compact) spacetimes have no conformal/Weyl (trace) anomalies...

$$\langle T^{\mu}_{\mu} \rangle^{d=2n+1} = 0, \quad n = 1, 2, \ldots$$

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$$\langle T^{\mu}_{\mu}\rangle^{d=2n+1} = \frac{2\delta(z)}{(d-1)!\,\mathrm{Vol}[\mathrm{S}^{d-1}]} \times \left[\sum_j b_j\, \mathrm{I}_j + (-1)^{(d-1)/2} a_d\, \mathring{E}_{d-1}\right], \quad n = 1, 2, \ldots$$

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where E_d, $\mathring{E}_{d-1}$ are the bulk/boundary Euler densities, and $E^{(\mathrm{bry})}$ the boundary term of the Euler characteristic... K_{pq} is the boundary extrinsic curvature, and h_{pq} is the induced metric on the boundary... dimensionalities $d = 5, 6$ not fully classified as of now (no nontrivial CFTs in $d > 6$)...

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$$\hat{K}_{pq} \equiv K_{pq} - \frac{h_{pq}}{d-1}K, \qquad \mathrm{tr}\hat{K}^2 \equiv \mathrm{tr}K^2 - \frac{1}{2}K^2, \qquad \mathrm{tr}\hat{K}^3 \equiv \mathrm{tr}K^3 - K\mathrm{tr}K^2 + \frac{2}{9}K^3$$

$$E_4 = \frac{1}{4}\delta^{\mu\nu\rho\sigma}_{\alpha\beta\gamma\delta} R^{\alpha\beta}_{\mu\nu} R^{\gamma\delta}_{\rho\sigma}, \qquad E^{(\mathrm{bry})}_4 = -4\delta^{stw}_{pqr} K^p_s \left(\frac{1}{2}R^{qr}_{tw} + \frac{2}{3}K^q_t K^r_w\right)$$

$$h^{\mu\nu}\hat{K}^{\rho\sigma} W_{\mu\nu\rho\sigma} = R^{\nu\rho\sigma}_{\mu} K^{\rho}_{\mu} n^{\nu} n^{\sigma} - \frac{1}{2}R_{\mu\nu}\left(n^{\mu}n^{\nu}K + K^{\mu\nu}\right) + \frac{1}{6}KR, \qquad h^{\mu\rho}\hat{K}^{\nu\sigma} W_{\mu\nu\rho\sigma} = -K^{pq} W_{npnq}.$$

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Anomaly coefficients in free theories

Before calculating the A & B anomaly coefficients for the D3-D5 dCFT, let us go through some results for codimension-1:

- In $d = 2$ the relation of the anomaly coefficient a to the central charge is $c = 12a$... For free scalar & Dirac fields:

$$a^{s=0} = a^{s=1/2} = \frac{1}{12} \qquad \text{(see e.g. Cardy, 2004).}$$

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$$a^{s=0}\big|_{\mathrm{D}}=-\frac{1}{96}, \qquad a^{s=0}\big|_{\mathrm{R}}=\frac{1}{96}, \qquad a^{s=1/2}=0, \qquad b^{s=0}\big|_{\mathrm{D/R}}=\frac{1}{64}, \qquad b^{s=1/2}=\frac{1}{32}.$$

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(see e.g. Birrell-Davies)... For the boundary charges of free fields, b_1 generally depends on the boundary conditions...

$$b_1^{s=0}\big|_{\mathrm{D}} = \frac{2}{35}, \qquad b_1^{s=0}\big|_{\mathrm{R}} = \frac{2}{45}, \qquad b_1^{s=1/2}\big|_{\mathrm{D/R}} = \frac{2}{7}, \qquad b_1^{s=1}\big|_{\mathrm{D/R}} = \frac{16}{35}$$

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Melmed (1988), Moss (1989)

whereas the (free field) boundary charge b_2 is independent of the BCs and proportional to the bulk central charge c:

$$b_2=8c.$$

Dowker-Schofield (1990)
Fursaev (2015), Solodukhin (2015)

Anomalies as observables (bulk)

All types (A, B, C) of anomaly coefficients show up in CFT and dCFT data... For the bulk charges,

- In $d = 2$, the central charge $c = 12a$ shows up in the two and three-point function of the (traceless) stress tensor:

$$\langle T(\mathfrak{z}_1)\, T(\mathfrak{z}_2)\rangle = \frac{c/2}{(\mathfrak{z}_1 - \mathfrak{z}_2)^4}, \qquad \langle T(\mathfrak{z}_1)\, T(\mathfrak{z}_2)\, T(\mathfrak{z}_3)\rangle = \frac{c}{(\mathfrak{z}_1 - \mathfrak{z}_2)^2 (\mathfrak{z}_2 - \mathfrak{z}_3)^2 (\mathfrak{z}_3 - \mathfrak{z}_1)^2},$$

where $T \equiv T_{\mathfrak{z}\mathfrak{z}}$, and $\mathfrak{z} \equiv x_1 + i x_2$, $\bar{\mathfrak{z}} \equiv x_1 - i x_2$ are the holomorphic/anti-holomorphic coordinates.

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  On the other hand, the (type A & C) conformal anomaly coefficients become:

$$c = \frac{N_0 + 3N_{1/2} + 12N_1}{120} = \frac{\pi^4 C_T}{40}, \qquad a = \frac{2N_0 + 11N_{1/2} + 124N_1}{720}$$

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  so that in the case of $U(N_c)$, $\mathcal{N} = 4$ SYM, all three coefficients turn out to be equal:

$$a = c = \frac{N_c^2}{4} = \frac{\pi^4 C_T}{40}.$$

Anomalies as observables (boundary)

The boundary charges show up in two and three-point functions of the displacement operator $\mathcal{D}$. In d dimensions,

$$\langle \mathcal{D}(\mathbf{x}_1)\,\mathcal{D}(\mathbf{x}_2)\rangle = \frac{C_{nn}}{\mathbf{x}_{12}^{2d}}, \qquad \langle \mathcal{D}(\mathbf{x}_1)\,\mathcal{D}(\mathbf{x}_2)\,\mathcal{D}(\mathbf{x}_3)\rangle = \frac{C_{nnn}}{\mathbf{x}_{12}^{d}\mathbf{x}_{23}^{d}\mathbf{x}_{31}^{d}}.$$

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It can be shown that the single 3d B-type anomaly coefficient and the two 4d B-type anomaly coefficients are given by:

$$b = \frac{\pi^2}{8}c_{nn}, \qquad b_1 = \frac{2\pi^3}{35}c_{nnn}, \qquad b_2 = \frac{2\pi^4}{15}c_{nn},$$

whereas there is no known relation for the 3d A-type anomaly coefficient a... Interestingly, the displacement operator computations confirm the (old) heat kernel results...

The D3-D5 stress tensor

Let us now compute the anomaly coefficients for the (codimension-1) dCFT that is dual to the D3-D5 probe-brane system... Because we are in 4d, there are 4 of them: the bulk charges c & a and the boundary charges b_1 & b_2...

Start off from the Lagrangian of $\mathcal{N} = 4$ SYM...

$$\mathcal{L}_{\mathcal{N}=4} = \frac{2}{g_{\text{YM}}^2} \cdot \text{tr}\Bigg\{ -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{2} \left(D_\mu \varphi_i\right)^2 + i\, \bar{\psi}_\alpha \not{D} \psi_\alpha + \frac{1}{4} \left[\varphi_i, \varphi_j\right]^2 + \\ + \sum_{i=1}^{3} G^i_{\alpha\beta} \bar{\psi}_\alpha \left[\varphi_i, \psi_\beta\right] + \sum_{i=4}^{6} G^i_{\alpha\beta} \bar{\psi}_\alpha \gamma_5 \left[\varphi_i, \psi_\beta\right] \Bigg\}.$$

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$$T_{\mu\nu} = \frac{\partial \mathcal{L}}{\partial \partial^\mu A_\rho} \partial_\nu A_\rho + \frac{\partial \mathcal{L}}{\partial \partial^\mu \varphi_i} \partial_\nu \varphi_i + \frac{\partial \mathcal{L}}{\partial \partial^\mu \bar{\psi}_\alpha} \partial_\nu \bar{\psi}_\alpha + \frac{\partial \mathcal{L}}{\partial \partial^\mu \psi_\alpha} \partial_\nu \psi_\alpha - g_{\mu\nu} \mathcal{L}.$$

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$$\Theta_{\mu\nu} = \frac{2}{g_{\text{YM}}^2} \cdot \text{tr}\left\{ -F_{\mu}{}^{\varrho} F_{\nu\varrho} - \frac{2}{3}\left(D_\mu\varphi_i\right)\left(D_\nu\varphi_i\right) + \frac{1}{3}\,\varphi_i\, D_{(\mu} D_{\nu)}\varphi_i + \frac{i}{2}\,\bar{\psi}_\alpha \gamma_{(\mu} \overleftrightarrow{D}_{\nu)} \psi_\alpha \right\} - g_{\mu\nu}\Lambda$$

$$\Lambda \equiv \frac{2}{g_{\text{YM}}^2} \cdot \text{tr}\left\{ -\frac{1}{4}\, F_{\mu\nu}F^{\mu\nu} - \frac{1}{6}\left(D_\mu\varphi_i\right)^2 - \frac{1}{12}\left[\varphi_i, \varphi_j\right]^2 \right\}, \qquad a_{(\mu\nu)} \equiv \frac{1}{2}\left(a_{\mu\nu} + a_{\nu\mu}\right).$$

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$$\langle \Theta_{\mu\nu}\left(\mathrm{x}_1\right) \Theta_{\rho\sigma}\left(\mathrm{x}_2\right)\rangle = \frac{640c}{\pi^4 \mathrm{x}_{12}^8} \cdot I_{\mu\nu\rho\sigma}\left(\mathrm{x}_1 - \mathrm{x}_2\right), \qquad c = \frac{N_c^2}{4},$$

which is found by Wick-contracting the perturbed fields with the $\mathcal{N}=4$ SYM Feynman rules (2 contractions for the LO)...

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To compute the defect anomaly coefficients, we will need only the scalar part of the (improved) stress tensor (since only scalars acquire vevs):

$$\Theta_{\mu\nu(\text{scalars})} = \frac{2}{g_{\text{YM}}^2} \cdot \text{tr}\Big\{-\frac{2}{3}\left(\partial_\mu\varphi_i\right)\left(\partial_\nu\varphi_i\right) + \frac{1}{3}\,\varphi_i\left(\partial_\mu\partial_\nu\varphi_i\right) + \frac{1}{6}\, g_{\mu\nu}\left[\left(\partial_\varrho\varphi_i\right)^2 + \frac{1}{2}\left[\varphi_i, \varphi_j\right]^2\right]\Big\}.$$

Stress tensor two-point function

Plugging the fuzzy funnel solution for the D3-D5 interface, we find that the stress tensor one-point function vanishes:

$$\langle \Theta_{\mu\nu}(\mathrm{x}) \rangle = 0,$$

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to lowest order in perturbation theory, as it should for a codimension-1 defect (McAvity-Osborn 1993 & 1995)...

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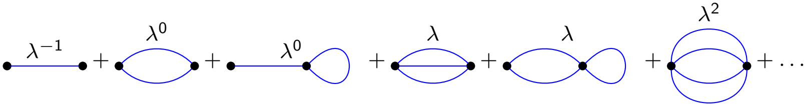
The LO contribution (order λ^{-1}) to the (connected) stress tensor two-point function consists of a single Wick contraction:

$$\langle \Theta_{\mu\nu}(\mathrm{x}_1)\, \Theta_{\rho\sigma}(\mathrm{x}_2) \rangle =$$



By expanding the $\mathcal{N} = 4$ fields around the fuzzy funnel solution of the D3-D5 interface we find:

$$\Theta^{(1)}_{\mu\nu}(x) = \frac{1}{g^2_{\mathrm{YM}}} \frac{4}{3z^2} \cdot \mathrm{tr}\left\{ \left(\frac{1}{z} \left(n_\mu n_\nu - g_{\mu\nu} \right) \tilde{\varphi}_i + n_\mu \partial_\nu \tilde{\varphi}_i + n_\nu \partial_\mu \tilde{\varphi}_i - \frac{g_{\mu\nu}}{2} \partial_3 \tilde{\varphi}_i + \frac{z}{2} \partial_\mu \partial_\nu \tilde{\varphi}_i \right) t_i \right\}.$$

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contracting with the propagator of the D3-D5 dCFT (Buhl-Mortensen, de Leeuw, Ipsen, Kristjansen, Wilhelm, 2016)...

$$X_\mu \equiv \mathrm{z}_1 \cdot \frac{\upsilon}{\xi} \frac{\partial \xi}{\partial \mathrm{x}_1^\mu} = \upsilon \left(\frac{2\mathrm{z}_1}{\mathrm{x}_{12}^2} (\mathrm{x}_{1\mu} - \mathrm{x}_{2\mu}) - n_\mu \right), \qquad X'_\rho \equiv \mathrm{z}_2 \cdot \frac{\upsilon}{\xi} \frac{\partial \xi}{\partial \mathrm{x}_2^\rho} = -\upsilon \left(\frac{2\mathrm{z}_2}{\mathrm{x}_{12}^2} (\mathrm{x}_{1\rho} - \mathrm{x}_{2\rho}) + n_\rho \right).$$

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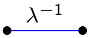

$$= \langle \Theta^{(1)}_{\mu\nu}(\mathrm{x}_1)\, \Theta^{(1)}_{\rho\sigma}(\mathrm{x}_2) \rangle = \frac{1}{\mathrm{x}_{12}^8} \cdot \Bigg\{ \left(X_\mu X_\nu - \frac{g_{\mu\nu}}{4} \right) \left(X'_\rho X'_\sigma - \frac{g_{\rho\sigma}}{4} \right) A(\upsilon) + \Big(X_\mu X'_\rho I_{\nu\sigma} + X_\mu X'_\sigma I_{\nu\rho} +$$

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$$A(\upsilon) = 4\gamma \left(6\upsilon^6 + 3\upsilon^4 + \upsilon^2\right), \quad B(\upsilon) = -\gamma \left(3\upsilon^6 - \upsilon^4 - 2\upsilon^2\right), \quad C(\upsilon) = \gamma \upsilon^2 \left(\upsilon^2 - 1\right)^2,$$

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which is valid for $k \geq 2$, while we have also defined,

$$\gamma \equiv \frac{32 c_k N_c}{9\pi^2 \lambda}, \qquad c_k \equiv \frac{k\left(k^2 - 1\right)}{4}, \qquad \xi \equiv \frac{\mathrm{x}_{12}^2}{4 \mathrm{z}_1 \mathrm{z}_2}, \qquad \upsilon^2 \equiv \frac{\xi}{1 + \xi}, \qquad \lambda \equiv g_{\mathrm{YM}}^2 N_c .$$

b_2 anomaly coefficient: D3-D5

As we have already mentioned, the b_2 coefficient can be read off the two-point function of the displacement operator $\mathcal{D}$:

$$\langle \mathcal{D}(\mathbf{x}_1)\,\mathcal{D}(\mathbf{x}_2)\rangle = \frac{c_{nn}}{\mathbf{x}_{12}^8}, \qquad c_{nn} = \frac{15 b_2}{2\pi^4}.$$

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Integrating over the transverse coordinate z from 0^- to 0^+ (and using the conformal invariance of the defect) we find:

$$\mathcal{D}(\mathbf{x}) = \lim_{z\to 0+} \Theta_{33}(z,\mathbf{x}) - \lim_{z\to 0-} \Theta_{33}(z,\mathbf{x})\,.$$

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$$b_2 = \frac{8\pi^2 k\,(k^2-1)N_c}{3\lambda} \neq 8c = 0.$$

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$$\mathcal{D}(\mathbf{x}) = \lim_{\mathrm{z}\to 0+} \Theta_{33}(\mathrm{z},\mathbf{x}) - \lim_{\mathrm{z}\to 0-} \Theta_{33}(\mathrm{z},\mathbf{x}).$$

The two-point function of the displacement operator then becomes:

$$\left\langle \mathcal{D}^{(1)}(\mathbf{x}_1)\,\mathcal{D}^{(1)}(\mathbf{x}_2)\right\rangle = \lim_{\mathrm{z}_1,\mathrm{z}_2\to 0^+} \left\langle \Theta_{33}^{(1)}(\mathrm{z}_1,\mathbf{x}_1)\,\Theta_{33}^{(1)}(\mathrm{z}_2,\mathbf{x}_2)\right\rangle = \frac{c_{nn}}{\mathbf{x}_{12}^8}, \quad c_{nn} = \frac{20k\,(k^2-1)N_c}{\pi^2\lambda},$$

and the b_2 anomaly coefficient (one contraction) is given by

$$b_2 = \frac{8\pi^2 k\,(k^2-1)N_c}{3\lambda} \neq 8c = 0.$$

de Leeuw-Kristjansen-GL-Volk (2023)

Despite not verifying the free-theory relation $b_2 = 8c$ (at the level of one Wick contraction), the value of b_2 confirms

$$\{\alpha(0),\alpha(1)\} = \{C_T, c_{nn}\} \xrightarrow{d=4} \left\{\frac{640c}{\pi^4}, \frac{15b_2}{2\pi^4}\right\}, \quad \alpha(v) = \frac{d-1}{d^2}\cdot\left[(d-1)(A(v)+4B(v)) + dC(v)\right],$$

for $d=4$ at the level of a single Wick contraction... These expressions appeared in Herzog-Huang (2017)...

Subsection 3

D3-D7 anomaly coefficients

b_2 anomaly coefficient: D3-D7

To compute the anomaly coefficients for the D3-D7 system (both $SO(5)$ and $SO(3) \times SO(3)$), we plug the corresponding fuzzy funnel solutions into the expression for the stress tensor... We find that the one-point function vanishes:

$$\langle \Theta_{\mu\nu}(\mathrm{x}) \rangle = 0,$$

work in progress

to lowest order in perturbation theory, as it should for a codimension-1 defect (McAvity-Osborn 1993 & 1995)...

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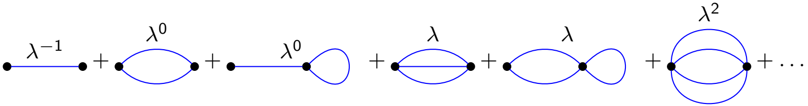
The LO contribution (order λ^{-1}) to the (connected) stress tensor two-point function consists of a single Wick contraction:

$$\langle \Theta_{\mu\nu}(\mathrm{x}_1)\, \Theta_{\rho\sigma}(\mathrm{x}_2) \rangle =$$



By expanding the $\mathcal{N} = 4$ fields around the fuzzy funnel solution of the D3-D7 interface we find:

$$\Theta^{(1)}_{\mu\nu}(x) = \frac{1}{g^2_{\mathrm{YM}}} \frac{4}{3z^2} \cdot \mathrm{tr}\left\{ \left(\frac{1}{z} (n_\mu n_\nu - g_{\mu\nu})\, \tilde{\varphi}_i + n_\mu \partial_\nu \tilde{\varphi}_i + n_\nu \partial_\mu \tilde{\varphi}_i - \frac{g_{\mu\nu}}{2} \partial_3 \tilde{\varphi}_i + \frac{z}{2} \partial_\mu \partial_\nu \tilde{\varphi}_i \right) \tau_i \right\}.$$

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$$\overset{\lambda^{-1}}{\bullet\!\!-\!\!\!-\!\!\!-\!\!\!-\!\!\bullet} = \langle \Theta^{(1)}_{\mu\nu}(\mathrm{x}_1)\, \Theta^{(1)}_{\rho\sigma}(\mathrm{x}_2) \rangle = \frac{1}{\mathrm{x}_{12}^8} \cdot \Bigg\{ \left(X_\mu X_\nu - \frac{g_{\mu\nu}}{4}\right)\left(X'_\rho X'_\sigma - \frac{g_{\rho\sigma}}{4}\right) A(\upsilon) + \Big(X_\mu X'_\rho I_{\nu\sigma} + X_\mu X'_\sigma I_{\nu\rho} +$$

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contracting with the propagator of the D3-D7 dCFT (Gimenez-Grau, Kristjansen, Volk, Wilhelm, 2019)...

$$X_\mu \equiv \mathrm{z}_1 \cdot \frac{\upsilon}{\xi} \frac{\partial \xi}{\partial \mathrm{x}_1^\mu} = \upsilon \left(\frac{2\mathrm{z}_1}{\mathrm{x}_{12}^2} (\mathrm{x}_{1\mu} - \mathrm{x}_{2\mu}) - n_\mu \right), \qquad X'_\rho \equiv \mathrm{z}_2 \cdot \frac{\upsilon}{\xi} \frac{\partial \xi}{\partial \mathrm{x}_2^\rho} = -\upsilon \left(\frac{2\mathrm{z}_2}{\mathrm{x}_{12}^2} (\mathrm{x}_{1\rho} - \mathrm{x}_{2\rho}) + n_\rho \right).$$

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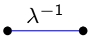

$$= \langle \Theta^{(1)}_{\mu\nu}(\mathrm{x}_1)\, \Theta^{(1)}_{\rho\sigma}(\mathrm{x}_2) \rangle = \frac{1}{\mathrm{x}_{12}^8} \cdot \Bigg\{ \left(X_\mu X_\nu - \frac{g_{\mu\nu}}{4} \right) \left(X'_\rho X'_\sigma - \frac{g_{\rho\sigma}}{4} \right) A(\upsilon) + \Big(X_\mu X'_\rho I_{\nu\sigma} + X_\mu X'_\sigma I_{\nu\rho} +$$

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$$A(\upsilon) = 4\gamma\left(6\upsilon^6 + 3\upsilon^4 + \upsilon^2\right), \quad B(\upsilon) = -\gamma\left(3\upsilon^6 - \upsilon^4 - 2\upsilon^2\right), \quad C(\upsilon) = \gamma\upsilon^2\left(\upsilon^2 - 1\right)^2,$$

$$\gamma \equiv \frac{32 c_k N_c}{9\pi^2\lambda}, \quad c_k \equiv \begin{cases} n(n+1)(n+2)(n+3)(n+4)/48, & SO(5) \\ k_1 k_2 \left(k_1^2 + k_2^2 - 2\right)/4, & SO(3) \times SO(3) \end{cases}, \quad \xi \equiv \frac{\mathrm{x}_{12}^2}{4 \mathrm{z}_1 \mathrm{z}_2}, \quad \upsilon^2 \equiv \frac{\xi}{1+\xi}.$$

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work in progress

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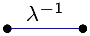

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The b_2 anomaly coefficient (at the level of a single Wick contraction) is found to be:

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Summary & outlook

We can summarize our results for the (LO) anomaly coefficients of the D3-D5 and D3-D7 holographic defects as follows:

$$c = 0, \quad b_2 = \frac{32\pi^2 c_k N_c}{3\lambda} \neq 8c = 0, \quad c_k \equiv \begin{cases} k\left(k^2-1\right)/4, \quad k \geq 2 & \text{D3-D5} \\ n(n+1)(n+2)(n+3)(n+4)/48, \quad n \geq 1 & \text{D3-D7 } [SO(5)] \\ k_1 k_2 \left(k_1^2 + k_2^2 - 2\right)/4, \quad k_{1,2} \geq 2 & \text{D3-D7 } [SO(3) \times SO(3)]. \end{cases}$$

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More results are underway...

- b_1 anomaly coefficient related to the stress tensor/displacement operator 3-point function ($b_1 = 2\pi^3 c_{nnn}/35$)...
- Crosscheck the D3-D5 results (analytically continued to $k=0$) from the 3d SCFT point of view...
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감사합니다!