Manifestly Covariant Formulat° of Worldline Particles

Why World Line Particles?

- Classical Analog of UIR
  Easier to handle
- Comparison with String
- Higher Spin gravity

I. Introduction

$$S = \int p_\mu dx^\mu + e(p^2 - m^2) = \int dt \left(\vec{p}\cdot\dot{\vec{x}} - \sqrt{\vec{p}^2 + m^2} \right)$$

$\Rightarrow$ $\vec{x}$: $d-1$ m.d.o.f (or $2(d-1)$-dim phase space)

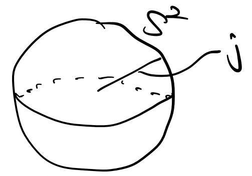
Scalar particle , Scalar UIR

- Scalar field: ∞ m.d.o.f ; 1. f.d.o.f
- Spinning field : n. f.d.o.f $= n \times \infty$ m.d.o.f

ex: lightcone $\phi_{\alpha}(t, \vec{x})$ — α: discrete, $\vec{x}$: conti

Spinning particle:
- $\vec{x}$ ($\vec{p}$) : $d-1$ m.d.o.f
- α (?) : 0 m.d.o.f
  (or n discrete d.o.f)
  (or n spin d.o.f)

What is classical description of spin d.o.f ?

Brezin : 

$\longleftrightarrow$ $(2j+1)$-dim rep of $SU(2)$

compact phase space

- Spherical harmonics on S^2
- only spin j survives ("spin projection")

Parametrizat' of S^2 :

- (θ, φ) : Not manifestly covariant
- $(\vec{x}, \vec{p})$: 6 instead of 2

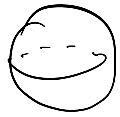
  Embedding Phase Space

  impose $\left[\begin{array}{l} 1 \text{ 1st class const} \\ 2 \text{ 2nd class const} \end{array}\right\}$ $\begin{array}{l} \vec{x}\cdot\vec{p} = 0 \\ \vec{p}^2 = j^2 \\ \vec{x}^2 = 1 \end{array}$

※ We can also use Grassmannian odd variables which also involve other d.o.f.

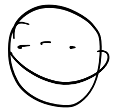
* Not to be confused with spinning top.

Relativistic Spinning Particles in Mink, (A)dS

 $\longleftrightarrow$ $(2j+1)$-rep of $SU(2)$

$\Downarrow$ $\qquad\qquad$ $\Downarrow$

Coadjoint orbits of $\left[\begin{array}{l} ISO(1,d-1) \\ SO(2,d) \\ SO(1,d+1) \end{array}\right.$ $\longleftrightarrow$ UIR of $\left[\begin{array}{l} ISO(1,d-1) \\ SO(2,d) \\ SO(1,d+1) \end{array}\right.$



$(\theta, \varphi) \rightarrow (\vec{x}, \vec{p})$ with constraints

$\Downarrow$

Coadjoint orbit intrinsic coordinate $\rightarrow$ Embedding phase space with constraints

"manifestly covariant formulat."

✲ We found all coadjoint orbits of $\left(\begin{array}{l} iso(1,d-1) \\ so(2,d) \\ so(1,d+1) \end{array} \right.$

and corresponding manifestly covariant formulations !

Not unique

- "Vectorial Model"
- "Twistor Model" $\Rightarrow$ Reproduce most of twistor particles and more.

For classical Lie group,

- matrix component as embedding space coord
- definition as constraints

II. Coadjoint orbits and geometric action

$\mathcal{O}_\phi = G/G_\phi$ symplectic

G : principal G_ϕ bundle over $\mathcal{O}_\phi$

- section $\sigma : \mathcal{O}_\phi \rightarrow G$

Symplectic 2-form $\omega = \sigma^* \langle \phi, g^{-1} d_G g \wedge g^{-1} d_G g \rangle = -d_0 \theta$

$$S = \int_{\gamma \subset \mathcal{O}_\phi} \theta = \int_{\sigma(\gamma) \subset G} \langle \phi, g^{-1} d_G g \rangle$$

$$Z = \int \mathcal{D}\gamma \, \exp\left(\frac{i}{\hbar} S\right)$$

σ independence of Z $\Rightarrow$ quantization of $\langle \phi, \underset{\Downarrow}{\underbrace{J_i}} \rangle$

$U(1) \subset G_\phi$

III. Classificat. of coadjoint orbits

$\Rightarrow$ Classificat. of ϕ

Poincaré

- $\phi = m p^0$, $\quad \mathfrak{g}_\phi = \underset{m}{\underline{\underline{\mathbb{R}}}} \oplus so(d-1)$

$$\frac{\dim \mathcal{O}_\phi}{2} = d-1$$

- $\phi = m p^0 + s J^{12}$, $\quad \mathfrak{g}_\phi = \underset{m}{\underline{\underline{\mathbb{R}}}} \oplus \underset{\text{(s) quantized}}{\underline{\underline{u(1)}}} \oplus so(d-3)$

$$\mathcal{O}_\phi = \frac{ISO(1,d-1)}{\mathbb{R} \times U(1) \times SO(d-3)}, \qquad \frac{\dim \mathcal{O}_\phi}{2} = d-1 + d-3$$

$$= \frac{ISO(1,d-1)}{\mathbb{R} \times SO(d-1)} \times \boxed{\frac{SO(d-1)}{SO(2) \times SO(d-3)}} \quad Gr_{\mathbb{R}}(2, d-1)$$

- $\phi = E p^+ + s J^{12}$, $\quad \mathfrak{g}_\phi = \left(\mathfrak{heis}_2 \oplus \mathfrak{u}(1)\right) \oplus \mathfrak{so}(d-4)$

$$\frac{\dim \mathcal{O}_\phi}{2} = d-1 + d-4 = 2(d-2)+1$$

AdS

$$p^A = J^{0'A}$$

- $\phi = m\, p^0$, $\quad \mathfrak{g}_\phi = \underbrace{\mathfrak{u}(1)}_{m \text{ quantized}} \oplus \mathfrak{so}(d-1)$

- $\phi = m\, p^0 + s J^{12}$, $\quad \mathfrak{g}_\phi = \underbrace{\mathfrak{u}(1)}_{m} \oplus \underbrace{\mathfrak{u}(1)}_{s} \oplus \mathfrak{so}(d-3)$

$$\mathcal{O}_\phi = \mathcal{O}_{\text{scalar}} \times Gr_{\mathbb{R}}(2, d-1)$$

- $\phi = s\, p^0 + s J^{12}$. $\quad \mathfrak{g}_\phi = \mathfrak{u}(1,1) \oplus \mathfrak{so}(d-3)$

- $\phi = \epsilon J^{++'}$ $\quad \left[\begin{array}{l} + = 0+1 \\ +' = 0'+2 \end{array}\right.$

$$\mathfrak{g}_\phi = \left(\mathfrak{sp}(2,\mathbb{R}) \oplus \mathfrak{so}(d-3)\right) \oplus \mathfrak{heis}_{2(d-3)}$$

$\Rightarrow$ Minimal Coadjoint orbit (Singleton)

$$\frac{\dim}{2} = d-2$$