S-matrix approach to AdS/CFT

Elementary level Introduction

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Very diverse audience

Physics

- Astrophysics (astro-ph new, recent, find) includes: Astrophysics of Galaxies; Cosmology and Nongalactic Astrophysics; Earth and Planetary Astrophysics; High Energy Astrophysical Phenomena; Instrumentation and Methods for Astrophysics; Solar and Stellar Astrophysics
 - General Relativity and Quantum Cosmology (gr-qc new, recent, find)
 - High Energy Physics Experiment (hep-ex new, recent, find)
 - High Energy Physics Lattice (hep-lat new, recent, find)
- High Energy Physics Phenomenology (hep-ph new, recent, find)
- High Energy Physics Theory (hep-th new, recent, find)
- Mathematical Physics (math-ph new, recent, find)
- Nonlinear Sciences (nlin new, recent, find) includes: Adaptation and Self-Organizing Systems; Cellular Automata and Lattice Gases; Chaotic Dynamics; Exactly Solvable and Integrable Systems; Pattern Formation and Solitons



- Abstract
- Unphysical
- Rigorous

- Physical

Goal

Can we find physical quantities from coupling constant?

F(g)

not so successful except perturbations

(Ex) magnetic moment of electron

• Anomalous moment:

$$-\operatorname{Exp.:} a_e \equiv \frac{g-2}{2} = 0.001 \ 159 \ 652 \ 180 \ 85(76)$$

- Should be determined by coupling constant

$$a_e = F(\alpha)$$
$$\alpha \equiv \frac{e^2}{4\pi\varepsilon_0\hbar c} = \frac{1}{137.035\ 999\ 074(44)}$$

Mag. Moment confirmed by perturbation theory

$$a_{e} = 0.5 \left(\frac{\alpha}{\pi}\right)$$

$$- 0.328 \ 478 \ 965 \ 579 \left(\frac{\alpha}{\pi}\right)^{2}$$

$$+ 1.181 \ 241 \ 456 \ 587 \left(\frac{\alpha}{\pi}\right)^{3}$$

$$- 1.509 \ 8(384) \left(\frac{\alpha}{\pi}\right)^{4}$$

What if $\alpha = 100?$

non-perturbative methods so far

- Classical: (ex) monopole, instanton
- Trivial theories: (ex) g=0 or infinity
- Trivial quantities: (ex) BPS
- Qualitative: (ex) holography
- Non-dynamical: (ex) localization

Real Non-perturbative theory?

Very difficult

Only way is EXACT SOLUTION

• INTEGRABILITY

SYMMETRY \rightarrow exact solution

S-matrix approach

S-matrix bootstrap

Infinite charges → completely elastic scattering



S-matrix enables exact computations of certain physical quantities

- Pole of S \rightarrow bound state
- (ex) delta-function potential in QM

– Bound state energy: $E = -\frac{m\alpha}{2\hbar^2}$

– Scattering amplitudes: (E>0)

$$r(p) = \frac{i\beta}{1 - i\beta}, \quad t(p) = \frac{i\beta}{1 - i\beta}, \quad \beta \equiv \frac{m\alpha}{\hbar p}$$

Pole at $p = i\frac{m\alpha}{\hbar} \quad \dots \rightarrow E = -\frac{m\alpha^2}{2\hbar^2}$

S-matrix Bootstrap



(ex) 2d Ising model with magnetic field

• S-matrices by Zamolodchikov (1988)

Mass spectrum from poles of S-matrices

 $m_1 = (4.40490857...)|B|^{5/13},$ $m_2 = 2m_1 \cos \frac{\pi}{5}, \dots \rightarrow \frac{\sqrt{5}+1}{2}$ $m_3 = 2m_1 \cos\frac{\pi}{30},$ Golden ratio $m_4 = 4m_1 \cos \frac{\pi}{5} \cos \frac{7\pi}{30},$ $m_5 = 4m_1 \cos \frac{\pi}{5} \cos \frac{2\pi}{15},$ $m_6 = 4m_1 \cos\frac{\pi}{5} \cos\frac{\pi}{30},$ $m_7 = 8m_1 \cos \frac{\pi}{5} \cos \frac{\pi}{5} \cos \frac{7\pi}{30},$ $m_8 = 8m_1 \cos\frac{\pi}{5} \cos\frac{\pi}{5} \cos\frac{2\pi}{15}$

Perron-Frobenius vector of Cartan matrix

 E_8 Lie algebra



R Coldea et al. Science 2010;327:177-180

(A and B) Energy scans at the zone center at 4.5 and 5 T observing two peaks, m1 and m2, at low energies.



R Coldea et al. Science 2010;327:177-180

AdS / CFT duality

Space with negative curvature



• Anti-de Sitter (AdS) space





AdS space

• Inside: 5 dim AdS₅ space



• Boundary: 4 dim space-time

Holography principle

Hologram



- Holography principle
 - String theory on AdS space = gauge theory on the boundary space-time
 - Quantum gravity effect $\leftarrow \rightarrow$ gauge interaction $\frac{1}{\lambda}$

AdS/CFT conjecture (Maldacena)

String theory on AdS₅ x S⁵



• 4 dim super Yang-Mills theory

Gauge theory

• Electromagnetism (Abelian gauge theory)

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} = \frac{1}{2} \left(\mathbf{E}^2 - \mathbf{B}^2 \right) \qquad F_{\mu\nu} = \partial_{\mu} A_{\nu} - \partial_{\nu} A_{\mu}$$

• Yang-Mills (non-Abelian gauge) theory:

 $A_{\mu} = N_c \times N_c$ Matrix

$$F_{\mu\nu} = \partial_{\mu}A_{\nu} - \partial_{\nu}A_{\mu} + ig[A_{\mu}, A_{\nu}]$$

$$\mathcal{L} = -\frac{1}{4} \operatorname{Tr} \left[F_{\mu\nu} F^{\mu\nu} \right] = \frac{1}{2} \sum_{a} \left(\mathbf{E}_{a}^{2} - \mathbf{B}_{a}^{2} \right)$$

Supersymmetric Yang-Mills theory

• Supersymmetry

- Scalar boson, fermion, gauge boson



$$-N=4 \text{ SYM} \qquad (A_{\mu}, \chi^{a}_{\alpha}, \Phi^{j}), \quad a = 1, \dots, 4; j = 1, \dots, 6$$
$$S = \frac{\text{Tr}}{g_{\text{YM}}^{2}} \int d^{4}x \left\{ -\frac{1}{4} F_{\mu\nu}^{2} + (D_{\mu}\Phi^{a})^{2} + \left[\Phi^{a}, \Phi^{b} \right]^{2} + \bar{\chi} \not D\chi - i\bar{\chi}\Gamma_{a}[\Phi^{a}, \chi] \right\}$$

- RG fixed point: scale invariance $\beta \equiv \mu \frac{\partial g_{\rm YM}}{\partial \mu} = -\frac{g_{\rm YM}^3}{16\pi^2} \left(\frac{11}{3} N_c - \frac{1}{6} \sum_i^{6N_c} C_i - \frac{1}{3} \sum_j^{8N_c} \tilde{C}_j \right) = 0$

AdS/CFT duality

Energy of string = Dimension of SYM



$$O(x) = \operatorname{Tr} \left[XYZF_{\mu\nu\chi} \chi^{\alpha}(D_{\mu}Y) \dots \right]$$
$$\langle O(x)O(0) \rangle = \frac{1}{|x|^{2\Delta}}$$
$$\overset{\wedge}{\underset{i}{\overset{\circ}{\atop}}}$$
Need non-perturbation

$$\lambda = g_{\rm YM}^2 N_c \equiv {\bf g}^2$$

Symmetry S-matrix **Physical Quantities**

<u>S-matrix</u>

• SYM : scattering of fields on the spin chain



• String side : scattering on the world sheet



• Symmetry : 4d conformal group psu(2,2|4)

• From symmetry to S-matrix

$$\left[\mathbf{S}(p_1, p_2), \left(\frac{\mathbb{L}_a^{\ b} \mid \mathbb{Q}_\alpha^{\ b}}{\mathbb{Q}_a^{\dagger\beta} \mid \mathbb{R}_\alpha^{\ \beta}} \right) \right] = \mathbf{0}$$

• $S: 16 \ge 16 \mod (a\beta)(\alpha\beta)$ ($a\beta$)($\alpha\beta$) ($a\beta$)($\alpha\beta$)($\alpha\beta$) ($a\beta$)($\alpha\beta$

$$\begin{split} S_{aa}^{aa} &= A, \quad S_{\alpha\alpha}^{\alpha\alpha} = D, \\ S_{ab}^{ab} &= \frac{1}{2}(A-B), \quad S_{ab}^{ba} = \frac{1}{2}(A+B), \\ S_{\alpha\beta}^{\alpha\beta} &= \frac{1}{2}(D-E), \quad S_{\alpha\beta}^{\beta\alpha} = \frac{1}{2}(D+E), \\ S_{ab}^{\alpha\beta} &= -\frac{1}{2}\epsilon_{ab}\epsilon^{\alpha\beta}C, \quad S_{\alpha\beta}^{ab} = -\frac{1}{2}\epsilon^{ab}\epsilon_{\alpha\beta}F, \\ S_{a\alpha}^{a\alpha} &= G, \quad S_{a\alpha}^{\alpha\alpha} = H, \quad S_{\alpha\alpha}^{\alpha\alpha} = K, \quad S_{\alpha\alpha}^{\alpha\alpha} = L \end{split}$$

$$\begin{split} A &= S_0 \frac{x_2^- - x_1^+}{x_2^+ - x_1^-} \frac{\eta_1 \eta_2}{\tilde{\eta}_1 \tilde{\eta}_2}, \\ B &= -S_0 \bigg[\frac{x_2^- - x_1^+}{x_2^+ - x_1^-} + 2 \frac{(x_1^- - x_1^+)(x_2^- - x_2^+)(x_2^- + x_1^+)}{(x_1^- - x_2^+)(x_1^- x_2^- - x_1^+ x_2^+)} \bigg] \frac{\eta_1 \eta_2}{\tilde{\eta}_1 \tilde{\eta}_2}, \\ C &= S_0 \frac{2ix_1^- x_2^-(x_1^+ - x_2^+)\eta_1 \eta_2}{x_1^+ x_2^+(x_1^- - x_2^+)(1 - x_1^- x_2^-)}, \quad D = -S_0, \\ E &= S_0 \bigg[1 - 2 \frac{(x_1^- - x_1^+)(x_2^- - x_2^+)(x_1^- + x_2^+)}{(x_1^- - x_2^+)(x_1^- x_2^- - x_1^+ x_2^+)} \bigg], \\ F &= S_0 \frac{2i(x_1^- - x_1^+)(x_2^- - x_2^+)(x_1^- - x_2^+)}{(x_1^- - x_2^+)(x_1^- - x_2^+)}, \\ G &= S_0 \frac{(x_2^- - x_1^-)}{(x_2^+ - x_1^-)\tilde{\eta}_1}, \quad H = S_0 \frac{(x_2^+ - x_2^-)}{(x_1^- - x_2^+)\tilde{\eta}_2}, \\ K &= S_0 \frac{(x_1^+ - x_1^-)\eta_2}{(x_1^- - x_2^+)\tilde{\eta}_1}, \quad L = S_0 \frac{(x_1^+ - x_2^+)\eta_2}{(x_1^- - x_2^+)\tilde{\eta}_2} \\ \eta_1 &= \eta(p_1)e^{ip_2/2}, \quad \eta_2 = \eta(p_2), \quad \tilde{\eta}_1 = \eta(p_1), \quad \tilde{\eta}_2 = \eta(p_2)e^{ip_1/2} \end{split}$$

beta-deformed SYM

$$V_{\beta} = \left| e^{i\pi\beta} ZX - e^{-i\pi\beta} XZ \right|^{2} + \left| e^{i\pi\beta} XY - e^{-i\pi\beta} YX \right|^{2} + \left| e^{i\pi\beta} YZ - e^{-i\pi\beta} ZY \right|^{2}$$

• N=1 super-CFT

• Dual to string theory on Lunin-Maldacena background $AdS_5 \times S_{\beta}^5$

S-matrix

[CA-Bajnok-Bombardelli-Nepomechie (2010a)]

• Drinfeld-twist by a constant matrix

$$S_{\beta}(p_1, p_2) = F_{\beta} \cdot S(p_1, p_2) \cdot F_{\beta}$$

Y-system

 $\ln Y_{N,M} = s \star \left[\ln(1 + Y_{N,M+1}) + \ln(1 + Y_{N,M-1}) \right] - s \star \left[\ln(1 + Y_{N+1,M}^{-1}) + \ln(1 + Y_{N-1,M}^{-1}) \right]$ M $\Delta = -\sum_{N=1}^{\infty} \int \frac{dp}{2\pi} \log(1 + Y_{N,0}) = \Delta(g)$

4-Loop su(2) Konishi

[Fiamberti, Santambrogio, Sieg, Zanon (2008)]

N=1 supergraphs



Wrapping diagrams



$$\begin{split} \gamma &= 4 + g^2 \gamma_1 + g^4 \gamma_2 + g^6 \gamma_3 + g^8 \gamma_4 + \dots \\ \gamma_1 &= 6(1+\delta) \\ \gamma_2 &= -\frac{3}{\delta} - 15 - 21\delta - 9\delta^2 \\ \gamma_3 &= -\frac{3}{4\delta^3} + \frac{153}{4\delta} + 114 + \frac{495}{4}\delta + 54\delta^2 + \frac{27}{4}\delta^3 \\ \gamma_4 &= -\frac{3}{8\delta^5} + \frac{33}{2\delta^3} - \frac{1701}{4\delta} - 1230 \\ &- \frac{2427}{2}\delta - 180\delta^2 + 162\delta^4 + \frac{2997}{8}\delta^3 \\ &+ \left(-\frac{9}{\delta} + 297 + 702\delta + 234\delta^2 - 405\delta^3 - 243\delta^4\right)\zeta(3) \\ &- 360\left(1+\delta\right)^2\zeta(5) \\ \delta &\equiv \frac{\sqrt{5+4}\cos(4\pi\beta)}{4} \end{split}$$

3

Results from Y-system [CA-Bajnok-Bombardelli-Nepomechie (2010b)]

$$\Delta = -\sum_{Q=1}^{\infty} \int_{-\infty}^{\infty} \frac{dq}{2\pi} \frac{4^L g^{2L}}{(Q^2 + q^2)^L} \sum_{j,j'} (-1)^{F_{(jj')}} \left[\mathcal{S}^{(Q1)}(q,p) \mathcal{S}^{(Q1)}(q,-p) \right]_{(jj')(11)}^{(jj')(11)}$$

 $\sum_{n=1}^{\infty} \left[\frac{f_1}{f_1} + \frac{f_2}{f_2} + f_2(Q) \right]$

After residue integrals

- (512 i (-5 i + 156 i Q² - 2120 i Q⁴ + 16512 i Q⁶ - 77952 i Q⁸ + 216064 i Q¹⁰ - 317440 i Q¹² + 196608 i Q¹⁴ - 32768 i Q¹⁶ - 160 Q u1 + 4352 Q³ u1 -50432 Q⁵ u1 + 321408 Q⁷ u1 - 1178624 Q⁹ u1 + 2396160 Q¹¹ u1 - 2490368 Q¹³ u1 + 1277952 Q¹⁵ u1 - 262144 Q¹⁷ u1 - 140 i u1² + 2490368 Q¹³ u1 + 1277952 Q¹⁵ u1 - 262144 Q¹⁷ u1 - 140 i u1² + 26214 Q¹⁷ u1 - 140 u1² u1² + 26214 Q¹⁷ u1 - 140 u1² + 26214 Q¹⁷ u1 - 26214 Q¹⁷ u1 - 26214 Q¹⁷ u1 - 26214 Q¹⁷ u1 - 26214 6036 i Q² u1² - 97280 i Q⁴ u1² + 813504 i Q⁶ u1² - 3904512 i Q⁸ u1² + 10729472 i Q¹⁰ u1² - 16007168 i Q¹² u1² + 12238848 i Q¹⁴ u1² -4456448 i Q¹⁶ u1² + 786432 i Q¹⁸ u1² - 3840 Q u1³ + 109184 Q³ u1³ - 1286656 Q⁵ u1³ + 8186112 Q⁷ u1³ - 29806080 Q⁹ u1³ + 60510208 Q¹¹ u1³ - $65617920 Q^{13} u1^3 + 38076416 Q^{15} u1^3 - 11403264 Q^{17} u1^3 + 1048576 Q^{19} u1^3 - 1700 i u1^4 + 85248 i Q^2 u1^4 - 1436928 i Q^4 u1^4 + 85248 i Q^2 u1^4 - 1436928 i Q^4 u1^4 + 1048576 Q^{19} u1^3 - 1700 i u1^4 + 1048576 Q^{19} u1^4 - 1436928 i Q^4 u1^4 + 1048576 Q^{19} u1^3 - 1700 i u1^4 - 1436928 i Q^4 u1^4 - 1$ 12094720 i Q⁶ u1⁴ - 57501696 i Q⁸ u1⁴ + 155193344 i Q¹⁰ u1⁴ - 229015552 i Q¹² u1⁴ + 182779904 i Q¹⁴ u1⁴ - 75235328 i Q¹⁶ u1⁴ + 13631488 i Q¹⁸ u1⁴ - 39040 Q u1⁵ + 1111040 Q³ u1⁵ - 12920832 Q⁵ u1⁵ + 80328704 Q⁷ u1⁵ - 284033024 Q⁹ u1⁵ + 555851776 Q¹¹ u1⁵ -584318976 Q¹³ u1⁵ + 322699264 Q¹⁵ u1⁵ - 83886080 Q¹⁷ u1⁵ + 2097152 Q¹⁹ u1⁵ - 11680 i u1⁶ + 614912 i Q² u1⁶ - 10256384 i Q⁴ u1⁶ + 83934720 i Q⁶ u1⁶ - 384442368 i Q⁸ u1⁶ + 984932352 i Q¹⁰ u1⁶ - 1352138752 i Q¹² u1⁶ + 976879616 i Q¹⁴ u1⁶ - 331350016 i Q¹⁶ u1⁶ + 27262976 i Q¹⁸ u1⁶ - 217600 Q u1⁷ + 5965824 Q³ u1⁷ - 66535424 Q⁵ u1⁷ + 393981952 Q⁷ u1⁷ - 1310375936 Q⁹ u1⁷ + 2346057728 Q¹¹ u1⁷ -2179596288 Q¹³ u1⁷ + 960495616 Q¹⁵ u1⁷ - 148897792 Q¹⁷ u1⁷ - 49600 i u1⁸ + 2532608 i Q² u1⁸ - 40226816 i Q⁴ u1⁸ + 310726656 i Q⁶ u1⁸ -1330298880 i Q⁸ u1⁸ + 3100934144 i Q¹⁰ u1⁸ - 3700162560 i Q¹² u1⁸ + 2134900736 i Q¹⁴ u1⁸ - 486539264 i Q¹⁶ u1⁸ + 8388608 i Q¹⁸ u1⁸ -716800 Q u1⁹ + 18268160 Q³ u1⁹ - 189792256 Q⁵ u1⁹ + 1041408000 Q⁷ u1⁹ - 3144482816 Q⁹ u1⁹ + 4859232256 Q¹¹ u1⁹ - 3649044480 Q¹³ u1⁹ + 1115684864 Q¹⁵ u1⁹ - 67108864 Q¹⁷ u1⁹ - 133120 i u1¹⁰ + 6116352 i Q² u1¹⁰ - 89391104 i Q⁴ u1¹⁰ + 633085952 i Q⁶ u1¹⁰ -2452226048 i Q⁸ u1¹⁰ + 4944297984 i Q¹⁰ u1¹⁰ - 4777312256 i Q¹² u1¹⁰ + 1935671296 i Q¹⁴ u1¹⁰ - 234881024 i Q¹⁶ u1¹⁰ - 1392640 Q u1¹¹ + $31883264 \text{ Q}^{3} \text{ u1}^{11} - 298713088 \text{ Q}^{5} \text{ u1}^{11} + 1471610880 \text{ Q}^{7} \text{ u1}^{11} - 3861774336 \text{ Q}^{9} \text{ u1}^{11} + 4783603712 \text{ Q}^{11} \text{ u1}^{11} - 2573205504 \text{ Q}^{13} \text{ u1}^{11} + 4783603712 \text{ Q}^{11} \text{ u1}^{11} - 2573205504 \text{ Q}^{13} \text{ u1}^{11} + 4783603712 \text{ Q}^{11} \text{ u1}^{11} - 2573205504 \text{ Q}^{13} \text{ u1}^{11} + 4783603712 \text{ Q}^{11} \text{ u1}^{11} - 2573205504 \text{ Q}^{13} \text{ u1}^{11} + 4783603712 \text{ Q}^{11} \text{ u1}^{11} - 2573205504 \text{ Q}^{13} \text{ u1}^{11} + 4783603712 \text{ Q}^{11} \text{ u1}^{11} - 2573205504 \text{ Q}^{13} \text{ u1}^{11} + 4783603712 \text{ Q}^{11} \text{ u1}^{11} +$ 469762048 Q¹⁵ u1¹¹ - 220160 i u1¹² + 8364032 i Q² u1¹² - 108068864 i Q⁴ u1¹² + 678658048 i Q⁶ u1¹² - 2279079936 i Q⁸ u1¹² + 3649568768 i O¹⁰ u1¹² - 2550136832 i O¹² u1¹² + 587202560 i O¹⁴ u1¹² - 1474560 O u1¹³ + 29360128 O³ u1¹³ - 237502464 O⁵ u1¹³ + 998375424 Q⁷ u1¹³ - 2085617664 Q⁹ u1¹³ + 1797259264 Q¹¹ u1¹³ - 469762048 Q¹³ u1¹³ - 204800 i u1¹⁴ + 5734400 i Q² u1¹⁴ -60817408 i O⁴ u1¹⁴ + 319029248 i O⁶ u1¹⁴ - 851443712 i O⁸ u1¹⁴ + 843055104 i O¹⁰ u1¹⁴ - 234881024 i O¹² u1¹⁴ - 655360 O u1¹⁵ + $11010048 \text{ Q}^{3} \text{ u1}^{15} - 71303168 \text{ Q}^{5} \text{ u1}^{15} + 222298112 \text{ Q}^{7} \text{ u1}^{15} - 234881024 \text{ Q}^{9} \text{ u1}^{15} + 67108864 \text{ Q}^{11} \text{ u1}^{15} - 81920 \text{ i u1}^{16} + 1376256 \text{ i Q}^{2} \text{ u1}^{16} - 81920 \text{ i u1}^{16} + 1376256 \text{ i Q}^{2} \text{ u1}^{16} - 81920 \text{ i u1}^{16} + 1376256 \text{ i Q}^{2} \text{ u1}^{16} - 81920 \text{ i u1}^{16} + 1376256 \text{ i Q}^{2} \text{ u1}^{16} - 81920 \text{ i u1}^{16} + 1376256 \text{ i Q}^{2} \text{ u1}^{16} - 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59152896 \text{ Q}^9 \text{ u1}^2 \text{ u2} + 117698560 \text{ Q}^{11} \text{ u1}^2 \text{ u2} - 59152896 \text{ Q}^9 \text{ u1}^2 \text{ u2} + 117698560 \text{ Q}^{11} \text{ u1}^2 \text{ u2} - 59152896 \text{ Q}^9 \text{ u1}^2 \text{ u2} + 117698560 \text{ Q}^{11} \text{ u1}^2 \text{ u2} - 59152896 \text{ Q}^9 \text{ u1}^2 \text{ u2} + 117698560 \text{ Q}^{11} \text{ u1}^2 \text{ u2} - 59152896 \text{ Q}^9 \text{ u1}^2 \text{ u2} + 117698560 \text{ Q}^{11} \text{ u1}^2 \text{ u2} - 59152896 \text{ Q}^9 \text{ u1}^2 \text{ u2} + 117698560 \text{ Q}^{11} \text{ u1}^2 \text{ u2} - 59152896 \text{ Q}^9 \text{ u1}^2 \text{ u2} + 117698560 \text{ Q}^{11} \text{ u1}^2 \text{ u2} - 59152896 \text{ Q}^9 \text{ u1}^2 \text{ u2} + 117698560 \text{ Q}^{11} \text{ u1}^2 \text{ u2} - 59152896 \text{ Q}^9 \text{ u1}^2 \text{ u2} + 117698560 \text{ Q}^{11} \text{ u1}^2 \text{ u2} - 59152896 \text{ Q}^9 \text{ u1}^2 \text{ u2} + 117698560 \text{ Q}^{11} \text{ u1}^2 \text{ u2} - 59152896 \text{ Q}^9 \text{ u1}^2 \text{ u2} + 117698560 \text{ Q}^{11} \text{ u1}^2 \text{ u2} - 59152896 \text{ Q}^9 \text{ u1}^2 \text{ u2} + 117698560 \text{ Q}^{11} \text{ u1}^2 \text{ u2} - 59152896 \text{ Q}^9 \text{ u1}^2 \text{ u2} + 117698560 \text{ Q}^{11} \text{ u1}^2 \text{ u2} - 59152896 \text{ Q}^9 \text{ u1}^2 \text{ u2} + 117698560 \text{ Q}^{11} \text{ u1}^2 \text{ u2} - 59152896 \text{ Q}^9 \text{ u1}^2 \text{ u2} + 117698560 \text{ Q}^{11} \text{ u1}^2 \text{ u2} - 59152896 \text{ Q}^9 \text{ u1}^2 \text{ u2} + 117698560 \text{ Q}^{11} \text{ u1}^2 \text{ u2} - 59152896 \text{ Q}^9 \text{ u1}^2 \text{ u2} + 117698560 \text{ Q}^{11} \text{ u1}^2 \text{ u2} - 59152896 \text{ Q}^9 \text{ u1}^2 \text{ u2} + 117698560 \text{ Q}^{11} \text{ u1}^2 \text{ u2} - 59152896 \text{ Q}^9 \text{ u1}^2 \text{ u2} + 117698560 \text{ Q}^{11} \text{ u2} + 117698560 \text{ u2} + 11$ $129744896 Q^{13} u1^2 u2 + 79364096 Q^{15} u1^2 u2 - 26869760 Q^{17} u1^2 u2 + 3145728 Q^{19} u1^2 u2 + 122688 i Q^2 u1^3 u2 - 2995968 i Q^4 u1^3 u2 + 122688 i Q^2 u1^3 u2 - 2995968 i Q^4 u1^3 u2 + 122688 i Q^2 u1^3 u2 - 2995968 i Q^4 u1^3 u2 + 122688 i Q^2 u1^3 u2 - 2995968 i Q^4 u1^3 u2 + 122688 i Q^2 u1^3 u2 - 2995968 i Q^4 u1^3 u2 + 122688 i Q^2 u1^3 u2 - 2995968 i Q^4 u1^3 u2 + 122688 i Q^2 u1^3 u2 - 2995968 i Q^4 u1^3 u2 + 122688 i Q^2 u1^3 u2 - 2995968 i Q^4 u1^3 u2 + 122688 i Q^4 u1^3 u2 + 122688 i Q^4 u1^3 u2 - 2995968 i Q^4 u1^3 u2 + 122688 i Q^4 u1^3 u2 +$ 29113856 i Q⁶ u1³ u2 - 143710208 i Q⁸ u1³ u2 + 382468096 i Q¹⁰ u1³ u2 - 560791552 i Q¹² u1³ u2 + 468844544 i Q¹⁴ u1³ u2 -220725248 i Q¹⁶ u1³ u2 + 46137344 i Q¹⁸ u1³ u2 - 54400 Q u1⁴ u2 + 2505216 Q³ u1⁴ u2 - 35850240 Q⁵ u1⁴ u2 + 241755136 Q⁷ u1⁴ u2 - $858062848 \, {\tt Q}^9 \, {\tt u1}^4 \, {\tt u2} + 1658478592 \, {\tt Q}^{11} \, {\tt u1}^4 \, {\tt u2} - 1806041088 \, {\tt Q}^{13} \, {\tt u1}^4 \, {\tt u2} + 1094189056 \, {\tt Q}^{15} \, {\tt u1}^4 \, {\tt u2} - 325058560 \, {\tt Q}^{17} \, {\tt u1}^4 \, {\tt u2} + 1094189056 \, {\tt Q}^{15} \, {\tt u1}^4 \, {\tt u2} - 325058560 \, {\tt Q}^{17} \, {\tt u1}^4 \, {\tt u2} + 1094189056 \, {\tt Q}^{15} \, {\tt u1}^4 \, {\tt u2} - 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325058560 \, {\tt u1}^{16} \, {\tt u2} \, {\tt$ 14680064 Q¹⁹ u1⁴ u2 + 1247360 i Q² u1⁵ u2 - 30472192 i Q⁴ u1⁵ u2 + 290317312 i Q⁶ u1⁵ u2 - 1384857600 i Q⁸ u1⁵ u2 + 3517972480 i Q¹⁰ u1⁵ u2 - 4901568512 i Q¹² u1⁵ u2 + 3774087168 i Q¹⁴ u1⁵ u2 - 1488977920 i Q¹⁶ u1⁵ u2 + 167772160 i Q¹⁸ u1⁵ u2 -373760 Q u1⁶ u2 + 18141184 Q³ u1⁶ u2 - 254623744 Q⁵ u1⁶ u2 + 1650380800 Q⁷ u1⁶ u2 - 5530009600 Q⁹ u1⁶ u2 + 9881518080 Q¹¹ u1⁶ u2 -9639165952 Q¹³ u1⁶ u2 + 4859101184 Q¹⁵ u1⁶ u2 - 945815552 Q¹⁷ u1⁶ u2 + 33554432 Q¹⁹ u1⁶ u2 + 6952960 i Q² u1⁷ u2 - 162570240 i Q⁴ u1⁷ u2 -1470644224 i Q⁶ u1⁷ u2 - 6562283520 i Q⁸ u1⁷ u2 + 15195176960 i Q¹⁰ u1⁷ u2 - 18733858816 i Q¹² u1⁷ u2 + 11924406272 i Q¹⁴ u1⁷ u2 -3439329280 i Q¹⁶ u1⁷ u2 + 301989888 i Q¹⁸ u1⁷ u2 - 1587200 Q u1⁸ u2 + 74530816 Q³ u1⁸ u2 - 984940544 Q⁵ u1⁸ u2 + 5928378368 Q⁷ u1⁸ u2 - 18003984384 Q⁹ u1⁸ u2 + 28163702784 Q¹¹ u1⁸ u2 - 22705864704 Q¹³ u1⁸ u2 + 8858370048 Q¹⁵ u1⁸ u2 -1325400064 Q¹⁷ u1⁸ u2 + 33554432 Q¹⁹ u1⁸ u2 + 22906880 i Q² u1⁹ u2 - 491487232 i Q⁴ u1⁹ u2 + 4084072448 i Q⁶ u1⁹ u2 -16451633152 i Q⁸ u1⁹ u2 + 33054523392 i Q¹⁰ u1⁹ u2 - 33852227584 i Q¹² u1⁹ u2 + 16949182464 i Q¹⁴ u1⁹ u2 - 3758096384 i Q¹⁶ u1⁹ u2 + 268435456 i Q¹⁸ u1⁹ u2 - 4259840 Q u1¹⁰ u2 + 178356224 Q³ u1¹⁰ u2 - 2139226112 Q⁵ u1¹⁰ u2 + 11541086208 Q⁷ u1¹⁰ u2 -

28059893760 i Q¹² u1¹¹ u2 + 10905190400 i Q¹⁴ u1¹¹ u2 - 1879048192 i Q¹⁶ u1¹¹ u2 - 7045120 Q u1¹² u2 + 239206400 Q³ u1¹² u2 -2496135168 Q⁵ u1¹² u2 + 11556880384 Q⁷ u1¹² u2 - 24384634880 Q⁹ u1¹² u2 + 24289214464 Q¹¹ u1¹² u2 - 11291066368 Q¹³ u1¹² u2 + 2348810240 Q¹⁵ u1¹² u2 + 47153152 i Q² u1¹³ u2 - 750256128 i Q⁴ u1¹³ u2 + 4593025024 i Q⁶ u1¹³ u2 - 12696158208 i Q⁸ u1¹³ u2 + 15523119104 i Q¹⁰ u1¹³ u2 - 8120172544 i Q¹² u1¹³ u2 + 1879048192 i Q¹⁴ u1¹³ u2 - 6553600 Q u1¹⁴ u2 + 157286400 Q³ u1¹⁴ u2 - $1317011456 \text{ } \text{0}^{5} \text{ } \text{u}^{14} \text{ } \text{u}^{2} + 4816109568 \text{ } \text{0}^{7} \text{ } \text{u}^{14} \text{ } \text{u}^{2} - 6861881344 \text{ } \text{0}^{9} \text{ } \text{u}^{14} \text{ } \text{u}^{2} + 3841982464 \text{ } \text{0}^{11} \text{ } \text{u}^{14} \text{ } \text{u}^{2} - 939524096 \text{ } \text{0}^{13} \text{ } \text{u}^{14} \text{ } \text{u}^{2} + 3841982464 \text{ } \text{0}^{11} \text{ } \text{u}^{14} \text{ } \text{u}^{2} + 3841982464 \text{ } \text{0}^{11} \text{ } \text{u}^{14} \text{ } \text{u}^{2} + 3841982464 \text{ } \text{0}^{11} \text{ } \text{u}^{14} \text{ } \text{u}^{2} + 3841982464 \text{ } \text{0}^{11} \text{ } \text{u}^{14} \text{ } \text{u}^{2} + 3841982464 \text{ } \text{0}^{11} \text{ } \text{u}^{14} \text{ } \text{u}^{2} + 3841982464 \text{ } \text{0}^{11} \text{ } \text{u}^{14} \text{ } \text{u}^{2} + 3841982464 \text{ } \text{0}^{11} \text{ } \text{u}^{14} \text{ } \text{u}^{2} + 3841982464 \text{ } \text{0}^{11} \text{ } \text{u}^{14} \text{ } \text{u}^{2} + 3841982464 \text{ } \text{0}^{11} \text{ } \text{u}^{14} \text{ } \text{u}^{2} + 3841982464 \text{ } \text{0}^{11} \text{ } \text{u}^{14} \text{ } \text{u}^{2} + 3841982464 \text{ } \text{0}^{11} \text{ } \text{u}^{14} \text{ } \text{u}^{2} + 3841982464 \text{ } \text{0}^{11} \text{ } \text{u}^{14} \text{ } \text{u}^{2} + 3841982464 \text{ } \text{0}^{11} \text{ } \text{u}^{14} \text{ } \text{u}^{2} + 3841982464 \text{ } \text{0}^{11} \text{ } \text{u}^{14} \text{ } \text{u}^{2} + 3841982464 \text{ } \text{0}^{11} \text{ } \text{u}^{14} \text{ } \text{u}^{2} + 3841982464 \text{ } \text{0}^{11} \text{ } \text{u}^{14} \text{ } \text{u}^{2} + 3841982464 \text{ } \text{0}^{11} \text{ } \text{u}^{14} \text{ } \text{u}^{2} + 3841982464 \text{ } \text{0}^{11} \text{ } \text{u}^{14} \text{ } \text{u}^{2} + 3841982464 \text{ } \text{0}^{11} \text{ } \text{u}^{14} \text{ } \text{u}^{2} + 3841982464 \text{ } \text{0}^{11} \text{ } \text{u}^{14} \text{ } \text{u}^{2} + 3841982464 \text{ } \text{0}^{11} \text{ } \text{u}^{14} \text{ } \text{u}^{2} + 3841982464 \text{ } \text{0}^{11} \text{ } \text{u}^{14} \text{ } \text{u}^{2} + 3841982464 \text{ } \text{0}^{11} \text{ } \text{u}^{14} \text{u}^{14} \text{ } \text{u}^{14} \text{ } \text{u}^{14} \text{ } \text{u$ 20971520 i Q² u1¹⁵ u2 - 268435456 i Q⁴ u1¹⁵ u2 + 1207959552 i Q⁶ u1¹⁵ u2 - 1862270976 i Q⁸ u1¹⁵ u2 + 1073741824 i Q¹⁰ u1¹⁵ u2 -268435456 i Q¹² u1¹⁵ u2 - 2621440 Q u1¹⁶ u2 + 33554432 Q³ u1¹⁶ u2 - 150994944 Q⁵ u1¹⁶ u2 + 232783872 Q⁷ u1¹⁶ u2 - 134217728 Q⁹ u1¹⁶ u2 + 33554432 Q¹¹ u1¹⁶ u2 - 140 i u2² + 6036 i Q² u2² - 97280 i Q⁴ u2² + 813504 i Q⁶ u2² - 3904512 i Q⁸ u2² + 10729472 i Q¹⁰ u2² -16007168 i Q¹² u2² + 12238848 i Q¹⁴ u2² - 4456448 i Q¹⁶ u2² + 786432 i Q¹⁸ u2² - 4480 Q u1 u2² + 174976 Q³ u1 u2² - 2408960 Q⁵ u1 u2² + $16281344 \text{ Q}^7 \text{ u1 u2}^2 - 59152896 \text{ Q}^9 \text{ u1 u2}^2 + 117698560 \text{ Q}^{11} \text{ u1 u2}^2 - 129744896 \text{ Q}^{13} \text{ u1 u2}^2 + 79364096 \text{ Q}^{15} \text{ u1 u2}^2 - 26869760 \text{ Q}^{17} \text{ u1 u2}^2 + 117698560 \text{ Q}^{11} \text{ u1 u2}^2 - 129744896 \text{ Q}^{13} \text{ u1 u2}^2 + 79364096 \text{ Q}^{15} \text{ u1 u2}^2 - 26869760 \text{ Q}^{17} \text{ u1 u2}^2 + 117698560 \text{ Q}^{11} \text{ u1 u2}^2 - 129744896 \text{ Q}^{13} \text{ u1 u2}^2 + 79364096 \text{ Q}^{15} \text{ u1 u2}^2 - 26869760 \text{ Q}^{17} \text{ u1 u2}^2 + 117698560 \text{ Q}^{11} \text{ u1 u2}^2 - 129744896 \text{ Q}^{13} \text{ u1 u2}^2 + 79364096 \text{ Q}^{15} \text{ u1 u2}^2 - 26869760 \text{ Q}^{17} \text{ u1 u2}^2 + 117698560 \text{ Q}^{11} \text{ u1 u2}^2 - 129744896 \text{ Q}^{13} \text{ u1 u2}^2 + 79364096 \text{ Q}^{15} \text{ u1 u2}^2 - 26869760 \text{ Q}^{17} \text{ u1 u2}^2 + 117698560 \text{ Q}^{11} \text{ u1 u2}^2 - 129744896 \text{ Q}^{13} \text{ u1 u2}^2 + 79364096 \text{ Q}^{15} \text{ u1 u2}^2 - 26869760 \text{ Q}^{17} \text{ u1 u2}^2 + 117698560 \text{ Q}^{11} \text{ u1 u2}^2 - 129744896 \text{ Q}^{13} \text{ u1 u2}^2 + 79364096 \text{ Q}^{15} \text{ u1 u2}^2 - 26869760 \text{ Q}^{17} \text{ u1 u2}^2 + 117698560 \text{ Q}^{11} \text{ u1 u2}^2 - 129744896 \text{ Q}^{13} \text{ u1 u2}^2 + 79364096 \text{ Q}^{15} \text{ u1 u2}^2 - 26869760 \text{ Q}^{17} \text{ u1 u2}^2 + 117698560 \text{ Q}^{11} \text{ u1 u2}^2 - 129744896 \text{ Q}^{13} \text{ u1 u2}^2 + 79364096 \text{ Q}^{15} \text{ u1 u2}^2 - 26869760 \text{ Q}^{17} \text{ u1 u2}^2 + 117698560 \text{ Q}^{11} \text{ u1 u2}^2 - 129744896 \text{ Q}^{13} \text{ u1 u2}^2 + 79364096 \text{ Q}^{15} \text{ u1 u2}^2 - 26869760 \text{ Q}^{17} \text{ u1 u2}^2 + 117698560 \text{ Q}^{11} \text{ u1 u2}^2 - 129744896 \text{ Q}^{13} \text{ u1 u2}^2 + 79364096 \text{ Q}^{15} \text{ u1 u2}^2 - 26869760 \text{ Q}^{17} \text{ u1 u2}^2 + 117698560 \text{ Q}^{11} \text{ u1 u2}^2 + 117698560 \text{ Q}^{11} \text{ u1 u2}^2 + 117698560 \text{ Q}^{13} \text{ u1 u2}^2 + 117698560 \text{ Q}^{15} \text{ u1 u2}^2 + 117698560 \text{ Q}^{11} \text{ u1 u2}^2 + 117698660$ 3145728 Q¹⁹ u1 u2² - 3880 i u1² u2² + 214272 i Q² u1² u2² - 4311296 i Q⁴ u1² u2² + 40286720 i Q⁶ u1² u2² - 196438016 i Q⁸ u1² u2² + $524476416 \text{ i} \text{ Q}^{10} \text{ u}^2 \text{ u}^2 - 783220736 \text{ i} \text{ Q}^{12} \text{ u}^2 \text{ u}^2 + 657063936 \text{ i} \text{ Q}^{14} \text{ u}^2 \text{ u}^2 - 309329920 \text{ i} \text{ Q}^{16} \text{ u}^2 \text{ u}^2 + 65011712 \text{ i} \text{ Q}^{18} \text{ u}^2 \text{ u}^2 - 309329920 \text{ i} \text{ Q}^{16} \text{ u}^2 \text{ u}^2 + 65011712 \text{ i} \text{ Q}^{18} \text{ u}^2 \text{ u}^2 - 309329920 \text{ i} \text{ Q}^{16} \text{ u}^2 \text{ u}^2 + 65011712 \text{ i} \text{ Q}^{18} \text{ u}^2 \text{ u}^2 + 65011712 \text{ i} \text{ Q}^{18} \text{ u}^2 \text{ u}^2 + 65011712 \text{ i} \text{ Q}^{18} \text{ u}^2 \text{ u}^2 + 65011712 \text{ i} \text{ u}^{18} \text{ u}^2 \text{ u}^2 + 65011712 \text{ i} \text{ u}^{18} \text{ u}^2 \text{ u}^2 + 65011712 \text{ i} \text{ u}^{18} \text{ u}^2 \text{ u}^2 + 65011712 \text{ i} \text{ u}^{18} \text{ u}^2 \text{ u}^2 + 65011712 \text{ i} \text{ u}^{18} \text{ u}^2 \text{ u}^2 + 65011712 \text{ i} \text{ u}^{18} \text{ u}^2 \text{ u}^2 + 65011712 \text{ i} \text{ u}^{18} \text{ u}^2 \text{ u}^2 + 65011712 \text{ i} \text{ u}^{18} \text{ u}^2 \text{ u}^2 \text{ u}^2 + 65011712 \text{ i} \text{ u}^{18} \text{ u}^2 \text{ u}^2 \text{ u}^2 + 65011712 \text{ i} \text{ u}^{18} \text{ u}^2 \text{ u}^2 \text{ u}^2 + 65011712 \text{ i} \text{ u}^{18} \text{ u}^2 + 65011712 \text{ u}^2 \text{ u}$ $3203268608 \text{ } \text{Q}^{13} \text{ } \text{u}^{1^{3}} \text{ } \text{u}^{2^{2}} + 2026110976 \text{ } \text{Q}^{15} \text{ } \text{u}^{1^{3}} \text{ } \text{u}^{2^{2}} - 631242752 \text{ } \text{Q}^{17} \text{ } \text{u}^{1^{3}} \text{ } \text{u}^{2^{2}} + 33554432 \text{ } \text{Q}^{19} \text{ } \text{u}^{1^{3}} \text{ } \text{u}^{2^{2}} - 46560 \text{ i } \text{u}^{1^{4}} \text{ } \text{u}^{2^{2}} + 33554432 \text{ } \text{Q}^{19} \text{ } \text{u}^{1^{3}} \text{ } \text{u}^{2^{2}} - 46560 \text{ i } \text{u}^{1^{4}} \text{ } \text{u}^{2^{2}} + 33554432 \text{ } \text{Q}^{19} \text{ } \text{u}^{1^{3}} \text{ } \text{u}^{2^{2}} - 46560 \text{ i } \text{u}^{1^{4}} \text{ } \text{u}^{2^{2}} + 33554432 \text{ } \text{Q}^{19} \text{ } \text{u}^{1^{3}} \text{ } \text{u}^{2^{2}} + 33554432 \text{ } \text{Q}^{19} \text{ } \text{u}^{1^{3}} \text{ } \text{u}^{2^{2}} + 33554432 \text{ } \text{Q}^{19} \text{ } \text{u}^{1^{3}} \text{ } \text{u}^{2^{2}} + 33554432 \text{ } \text{Q}^{19} \text{ } \text{u}^{1^{3}} \text{ } \text{u}^{2^{2}} + 33554432 \text{ } \text{Q}^{19} \text{ } \text{u}^{1^{3}} \text{ } \text{u}^{2^{2}} + 33554432 \text{ } \text{Q}^{19} \text{ } \text{u}^{1^{3}} \text{ } \text{u}^{2^{2}} + 33554432 \text{ } \text{Q}^{19} \text{ } \text{u}^{1^{3}} \text{ } \text{u}^{2^{2}} + 33554432 \text{ } \text{Q}^{19} \text{ } \text{u}^{1^{3}} \text{ } \text{u}^{2^{2}} + 33554432 \text{ } \text{u}^{1^{3}} \text{ } \text{u}^{2^{3}} + 33554432 \text{ } \text{u}^{2^{3}} \text{ } \text{u}^{2^{3}} \text{ } \text{u}^{2^{3}} \text{u}^{2^{3}} \text{ } \text{u}^{2^{3}}$ 2903680 i Q² u1⁴ u2² - 62765056 i Q⁴ u1⁴ u2² + 594636288 i Q⁶ u1⁴ u2² - 2844647424 i Q⁸ u1⁴ u2² + 7336697856 i Q¹⁰ u1⁴ u2² -10593501184 i Q¹² u1⁴ u2² + 8597929984 i Q¹⁴ u1⁴ u2² - 3594518528 i Q¹⁶ u1⁴ u2² + 450887680 i Q¹⁸ u1⁴ u2² - 1064960 Q u1⁵ u2² + 43145216 Q³ u1⁵ u2² - 6069698 $340 Q^{11} u1^5 u2^2 \sum_{20476928 \, i \, Q^2 \, u1^6 \, u2^2 - 445663}^{25783566336 \, Q^{13} \, u1^5 \, u2^2 + 1403} 8 \text{ more pages like this}$ 520 i u1⁶ u2² + 40281344 i Q¹⁰ u1⁶ u2² -57707855872 i Q¹² u1⁶ u2² + 40070071072 I y u1 u2 - 12402240704 I y u1 u2 + 1023410170 I y u1 u2² - 5836800 Q u1⁷ u2² + $86576726016 \, {\tt Q^{13}} \, {\tt u1^7} \, {\tt u2^2} + 36859543552 \, {\tt Q^{15}} \, {\tt u1^7} \, {\tt u2^2} - 5687476224 \, {\tt Q^{17}} \, {\tt u1^7} \, {\tt u2^2} + 67108864 \, {\tt Q^{19}} \, {\tt u1^7} \, {\tt u2^2} - 1318400 \, {\tt i} \, {\tt u1^8} \, {\tt u2^2} + 1318400 \, {\tt u1^8} \, {\tt u2^8} \, {\tt u2^8} \, {\tt u1^8} \, {\tt u2^8} \, {\tt u2^8} \, {\tt u1^8} \, {\tt u2^8} \, {\tt u2^8} \, {\tt u2^8} \, {\tt u1^8} \, {\tt u2^8} \, {$ 83460096 i Q² u1⁸ u2² - 1750466560 i Q⁴ u1⁸ u2² + 15009816576 i Q⁶ u1⁸ u2² - 61943316480 i Q⁸ u1⁸ u2² + 131372548096 i Q¹⁰ u1⁸ u2² -144929980416 i Q¹² u1⁸ u2² + 81040244736 i Q¹⁴ u1⁸ u2² - 19293798400 i Q¹⁶ u1⁸ u2² + 838860800 i Q¹⁸ u1⁸ u2² - 18841600 Q u1⁹ u2² + $712146944 \text{ Q}^{3} \text{ u1}^{9} \text{ u2}^{2} - 8872656896 \text{ Q}^{5} \text{ u1}^{9} \text{ u2}^{2} + 49515855872 \text{ Q}^{7} \text{ u1}^{9} \text{ u2}^{2} - 135757168640 \text{ Q}^{9} \text{ u1}^{9} \text{ u2}^{2} + 190264115200 \text{ Q}^{11} \text{ u1}^{9} \text{ u2}^{2} - 135757168640 \text{ Q}^{9} \text{ u1}^{9} \text{ u2}^{2} + 190264115200 \text{ Q}^{11} \text{ u1}^{9} \text{ u2}^{2} - 135757168640 \text{ Q}^{9} \text{ u1}^{9} \text{ u2}^{2} + 190264115200 \text{ Q}^{11} \text{ u1}^{9} \text{ u2}^{2} - 135757168640 \text{ Q}^{9} \text{ u1}^{9} \text{ u2}^{2} + 190264115200 \text{ Q}^{11} \text{ u1}^{9} \text{ u2}^{2} - 135757168640 \text{ Q}^{9} \text{ u1}^{9} \text{ u2}^{2} + 190264115200 \text{ Q}^{11} \text{ u1}^{9} \text{ u2}^{2} - 135757168640 \text{ Q}^{9} \text{ u1}^{9} \text{ u2}^{2} + 190264115200 \text{ Q}^{11} \text{ u1}^{9} \text{ u2}^{2} - 135757168640 \text{ Q}^{9} \text{ u1}^{9} \text{ u2}^{2} + 190264115200 \text{ Q}^{11} \text{ u1}^{9} \text{ u2}^{2} - 135757168640 \text{ Q}^{9} \text{ u1}^{9} \text{ u2}^{2} + 190264115200 \text{ Q}^{11} \text{ u1}^{9} \text{ u2}^{2} - 135757168640 \text{ Q}^{9} \text{ u1}^{9} \text{ u2}^{2} + 190264115200 \text{ Q}^{11} \text{ u1}^{9} \text{ u2}^{2} - 135757168640 \text{ Q}^{9} \text{ u1}^{9} \text{ u2}^{2} + 190264115200 \text{ Q}^{11} \text{ u1}^{9} \text{ u2}^{2} - 135757168640 \text{ Q}^{9} \text{ u1}^{9} \text{ u2}^{2} + 190264115200 \text{ Q}^{11} \text{ u1}^{9} \text{ u2}^{2} - 135757168640 \text{ Q}^{9} \text{ u1}^{9} \text{ u2}^{2} + 190264115200 \text{ Q}^{11} \text{ u1}^{9} \text{ u2}^{2} - 135757168640 \text{ Q}^{9} \text{ u1}^{9} \text{ u2}^{2} + 190264115200 \text{ Q}^{11} \text{ u1}^{9} \text{ u2}^{2} - 135757168640 \text{ Q}^{9} \text{ u1}^{9} \text{ u2}^{2} + 190264115200 \text{ Q}^{11} \text{ u1}^{9} \text{ u2}^{2} + 190264115200 \text{ U2}^{11} \text{ u1}^{9} \text{ u2}^{2} + 190264115200 \text{ U2}^{11} \text{ u1}^{9} \text{ u2}^{2} + 190264115200 \text{ u2}^{11} \text{ u1}^{9} \text{ u2}^{11} \text{ u1}^{9} \text{ u2}^{2} + 190264115200 \text{ u2}^{11} \text{ u1}^{9} \text{ u2}^{2} + 190264115200 \text{ u2}^{11} \text{ u1}^{9} \text{ u2}^{11} \text{ u2}^{11} \text{ u1}^{11} \text{ u1}^{11} \text{ u2}^{11} \text{ u1}^{11} \text{ u2}^{11} \text{ u1}^{11} \text{ u2}^{11} \text{ u2}^{11} \text{ u2}^{11} \text{ u1}^{11} \text{ u2}^{11} \text{ u2}^{11} \text{ u2}^{11} \text{ u1}^{11} \text{ u2}^{11} \text{ u2}^{11}$ $135780106240 Q^{13} u1^9 u2^2 + 45063602176 Q^{15} u1^9 u2^2 - 4362076160 Q^{17} u1^9 u2^2 - 3471360 i u1^{10} u2^2 + 202014720 i Q^2 u1^{10} u2^2 - 4362076160 Q^{17} u1^9 u2^2 - 3471360 i u1^{10} u2^2 + 202014720 i Q^2 u1^{10} u2^2 - 4362076160 Q^{17} u1^9 u2^2 - 3471360 i u1^{10} u2^2 + 202014720 i Q^2 u1^{10} u2^2 - 4362076160 Q^{17} u1^9 u2^2 - 3471360 i u1^{10} u2^2 + 202014720 i Q^2 u1^{10} u2^2 - 4362076160 Q^{17} u1^9 u2^2 - 3471360 i u1^{10} u2^2 + 202014720 i Q^2 u1^{10} u2^2 - 4362076160 Q^{17} u1^9 u2^2 - 3471360 i u1^{10} u2^2 + 202014720 i Q^2 u1^{10} u2^2 - 4362076160 Q^{17} u1^9 u2^2 - 3471360 i u1^{10} u2^2 + 202014720 i Q^2 u1^{10} u2^2 - 4362076160 Q^{17} u1^9 u2^2 - 3471360 i u1^{10} u2^2 + 202014720 i Q^2 u1^{10} u2^2 - 4362076160 Q^{17} u1^9 u2^2 - 3471360 i u1^{10} u2^2 + 202014720 i Q^2 u1^{10} u2^2 - 4362076160 Q^{17} u1^9 u2^2 - 3471360 i u1^{10} u2^2 + 202014720 i Q^2 u1^{10} u2^2 - 4362076160 Q^{17} u1^9 u2^2 - 3471360 i u1^{10} u2^2 + 202014720 i Q^2 u1^{10} u2^2 - 4362076160 Q^{17} u1^9 u2^2 - 3471360 i u1^{10} u2^2 + 202014720 i Q^2 u1^{10} u2^2 - 4362076160 Q^{17} u1^9 u2^2 - 3471360 i u1^{10} u2^2 + 202014720 i Q^2 u1^{10} u2^2 - 4362076160 Q^{17} u1^9 u2^2 - 3471360 i u1^{10} u2^2 + 202014720 i Q^2 u1^{10} u2^2 - 4362076160 Q^{17} u1^9 u2^2 - 3471360 i u1^{10} u2^2 + 202014720 i Q^2 u1^{10} u2^2 - 4362076160 Q^{17} u1^9 u2^2 - 3471360 i u1^{10} u2^2 + 202014720 i Q^2 u1^{10} u2^2 - 4362076160 Q^{17} u1^9 u2^2 - 3471360 i u1^{10} u2^2 + 202014720 i Q^2 u1^{10} u2^2 - 4362076160 Q^{17} u1^9 u2^2 - 3471360 i u1^{10} u2^2 + 4362076160 Q^{17} u2^2 + 4362076160 Q^{$ 3918004224 i Q⁴ u1¹⁰ u2² + 30297948160 i Q⁶ u1¹⁰ u2² - 109129236480 i Q⁸ u1¹⁰ u2² + 195700981760 i Q¹⁰ u1¹⁰ u2² -175187689472 i Q¹² u1¹⁰ u2² + 76822872064 i Q¹⁴ u1¹⁰ u2² - 12683575296 i Q¹⁶ u1¹⁰ u2² - 35717120 Q u1¹¹ u2² + 1265500160 Q³ u1¹¹ u2² - $13993246720 \, 9^{5} \, \mathrm{u1^{11}} \, \mathrm{u2^{2}} + 67825041408 \, 9^{7} \, \mathrm{u1^{11}} \, \mathrm{u2^{2}} - 156105703424 \, 9^{9} \, \mathrm{u1^{11}} \, \mathrm{u2^{2}} + 174785036288 \, 9^{11} \, \mathrm{u1^{11}} \, \mathrm{u2^{2}} - 97626619904 \, 9^{13} \, \mathrm{u1^{11}} \, \mathrm{u2^{2}} + 174785036288 \, 9^{11} \, \mathrm{u1^{11}} \, \mathrm{u2^{2}} - 97626619904 \, 9^{13} \, \mathrm{u1^{11}} \, \mathrm{u2^{2}} + 174785036288 \, 9^{11} \, \mathrm{u1^{11}} \, \mathrm{u2^{2}} - 97626619904 \, 9^{13} \, \mathrm{u1^{11}} \, \mathrm{u2^{2}} + 174785036288 \, 9^{11} \, \mathrm{u1^{11}} \, \mathrm{u2^{2}} - 97626619904 \, 9^{13} \, \mathrm{u1^{11}} \, \mathrm{u2^{2}} + 174785036288 \, 9^{11} \, \mathrm{u1^{11}} \, \mathrm{u2^{2}} - 97626619904 \, 9^{13} \, \mathrm{u1^{11}} \, \mathrm{u2^{2}} + 174785036288 \, 9^{11} \, \mathrm{u1^{11}} \, \mathrm{u2^{2}} - 97626619904 \, 9^{13} \, \mathrm{u1^{11}} \, \mathrm{u2^{2}} + 174785036288 \, 9^{11} \, \mathrm{u1^{11}} \, \mathrm{u2^{2}} - 97626619904 \, 9^{13} \, \mathrm{u1^{11}} \, \mathrm{u2^{2}} + 174785036288 \, 9^{11} \, \mathrm{u1^{11}} \, \mathrm{u2^{2}} - 97626619904 \, 9^{13} \, \mathrm{u1^{11}} \, \mathrm{u2^{2}} + 174785036288 \, 9^{11} \, \mathrm{u1^{11}} \, \mathrm{u2^{2}} - 97626619904 \, 9^{13} \, \mathrm{u1^{11}} \, \mathrm{u2^{2}} + 174785036288 \, 9^{11} \, \mathrm{u1^{11}} \, \mathrm{u2^{2}} + 174785036288 \, \mathrm{u2^{11}} \, \mathrm{u2^{11}}$ 23018340352 Q¹⁵ u1¹¹ u2² - 5611520 i u1¹² u2² + 282558464 i Q² u1¹² u2² - 4798283776 i Q⁴ u1¹² u2² + 32089702400 i Q⁶ u1¹² u2² -95158272000 i Q⁸ u1¹² u2² + 134215630848 i Q¹⁰ u1¹² u2² - 91838480384 i Q¹² u1¹² u2² + 27246198784 i Q¹⁴ u1¹² u2² - 36700160 Q u1¹³ u2² + 21139292160 Q¹³ u1¹³ u2² - 5079040 i u1¹⁴ u2² + 206831616 i Q² u1¹⁴ u2² - 2758279168 i Q⁴ u1¹⁴ u2² + 14705229824 i Q⁶ u1¹⁴ u2² -32015122432 i Q⁸ u1¹⁴ u2² + 28286386176 i Q¹⁰ u1¹⁴ u2² - 10401873920 i Q¹² u1¹⁴ u2² - 15728640 Q u1¹⁵ u2² + 476053504 Q³ u1¹⁵ u2² - $3422552064 \text{ } \text{Q}^{5} \text{ } \text{u1}^{15} \text{ } \text{u2}^{2} + 8405385216 \text{ } \text{Q}^{7} \text{ } \text{u1}^{15} \text{ } \text{u2}^{2} - 7784628224 \text{ } \text{Q}^{9} \text{ } \text{u1}^{15} \text{ } \text{u2}^{2} + 2952790016 \text{ } \text{Q}^{11} \text{ } \text{u1}^{15} \text{ } \text{u2}^{2} - 1966080 \text{ i } \text{u1}^{16} \text{ } \text{u2}^{2} + 2952790016 \text{ } \text{Q}^{11} \text{ } \text{u1}^{15} \text{ } \text{u2}^{2} + 2952790016 \text{ } \text{Q}^{11} \text{ } \text{u1}^{15} \text{ } \text{u2}^{2} + 2952790016 \text{ } \text{Q}^{11} \text{ } \text{u1}^{15} \text{ } \text{u2}^{2} + 2952790016 \text{ } \text{Q}^{11} \text{ } \text{u1}^{15} \text{ } \text{u2}^{2} + 2952790016 \text{ } \text{Q}^{11} \text{ } \text{u1}^{15} \text{ } \text{u2}^{2} + 2952790016 \text{ } \text{Q}^{11} \text{ } \text{u1}^{15} \text{ } \text{u2}^{2} + 2952790016 \text{ } \text{Q}^{11} \text{ } \text{u1}^{15} \text{ } \text{u2}^{2} + 2952790016 \text{ } \text{Q}^{11} \text{ } \text{u1}^{15} \text{ } \text{u2}^{2} + 2952790016 \text{ } \text{Q}^{11} \text{ } \text{u1}^{15} \text{ } \text{u2}^{2} + 2952790016 \text{ } \text{Q}^{11} \text{ } \text{u1}^{15} \text{ } \text{u2}^{2} + 2952790016 \text{ } \text{Q}^{11} \text{ } \text{u1}^{15} \text{ } \text{u2}^{2} + 2952790016 \text{ } \text{Q}^{11} \text{ } \text{u1}^{15} \text{ } \text{u2}^{2} + 2952790016 \text{ } \text{Q}^{11} \text{ } \text{u1}^{15} \text{ } \text{u2}^{2} + 2952790016 \text{ } \text{Q}^{11} \text{ } \text{u1}^{15} \text{ } \text{u2}^{2} + 2952790016 \text{ } \text{Q}^{11} \text{ } \text{u1}^{15} \text{ } \text{u2}^{2} + 2952790016 \text{ } \text{Q}^{11} \text{ } \text{u1}^{15} \text{ } \text{u2}^{2} + 2952790016 \text{ } \text{Q}^{11} \text{ } \text{u1}^{15} \text{ } \text{u2}^{2} + 2952790016 \text{ } \text{Q}^{11} \text{ } \text{u1}^{15} \text{ } \text{u2}^{2} + 2952790016 \text{ } \text{Q}^{11} \text{ } \text{u1}^{15} \text{ } \text{u2}^{2} + 2952790016 \text{ } \text{Q}^{11} \text{ } \text{u1}^{15} \text{ } \text{u2}^{2} + 2952790016 \text{ } \text{Q}^{11} \text{ } \text{u1}^{15} \text{ } \text{u2}^{2} + 2952790016 \text{ } \text{Q}^{11} \text{ } \text{u1}^{15} \text{ } \text{u2}^{2} + 2952790016 \text{ } \text{Q}^{11} \text{ } \text{u1}^{15} \text{ } \text{u2}^{2} + 2952790016 \text{ } \text{U}^{11} \text{ } \text{u1}^{11} \text{ } \text{u2}^{11} \text$ $59506688 \text{ i} \text{ 0}^2 \text{ u1}^{16} \text{ u2}^2 - 427819008 \text{ i} \text{ 0}^4 \text{ u1}^{16} \text{ u2}^2 + 1050673152 \text{ i} \text{ 0}^6 \text{ u1}^{16} \text{ u2}^2 - 973078528 \text{ i} \text{ 0}^8 \text{ u1}^{16} \text{ u2}^2 + 369098752 \text{ i} \text{ 0}^{10} \text{ u1}^{16} \text{ u2}^2 - 973078528 \text{ i} \text{ 0}^8 \text{ u1}^{16} \text{ u2}^2 + 369098752 \text{ i} \text{ 0}^{10} \text{ u1}^{16} \text{ u2}^2 - 973078528 \text{ i} \text{ 0}^8 \text{ u1}^{16} \text{ u2}^2 + 369098752 \text{ i} \text{ 0}^{10} \text{ u1}^{16} \text{ u2}^2 - 973078528 \text{ i} \text{ 0}^8 \text{ u1}^{16} \text{ u2}^2 + 369098752 \text{ i} \text{ 0}^{10} \text{ u1}^{16} \text{ u2}^2 - 973078528 \text{ i} \text{ 0}^8 \text{ u1}^{16} \text{ u2}^2 + 369098752 \text{ i} \text{ 0}^{10} \text{ u1}^{16} \text{ u2}^2 - 973078528 \text{ i} \text{ 0}^8 \text{ u1}^{16} \text{ u2}^2 + 369098752 \text{ i} \text{ 0}^{10} \text{ u1}^{16} \text{ u2}^2 - 973078528 \text{ i} \text{ 0}^8 \text{ u1}^{16} \text{ u2}^2 + 369098752 \text{ i} \text{ 0}^{10} \text{ u1}^{16} \text{ u2}^2 - 973078528 \text{ i} \text{ 0}^8 \text{ u1}^{16} \text{ u2}^2 + 369098752 \text{ i} \text{ 0}^{10} \text{ u1}^{16} \text{ u2}^2 - 973078528 \text{ i} \text{ 0}^8 \text{ u1}^{16} \text{ u2}^2 + 369098752 \text{ i} \text{ 0}^{10} \text{ u1}^{16} \text{ u2}^2 - 973078528 \text{ i} \text{ 0}^8 \text{ u1}^{16} \text{ u2}^2 + 369098752 \text{ i} \text{ 0}^{10} \text{ u1}^{16} \text{ u2}^2 - 973078528 \text{ i} \text{ 0}^8 \text{ u1}^{16} \text{ u2}^2 + 369098752 \text{ i} \text{ 0}^{10} \text{ u1}^{16} \text{ u2}^2 - 973078528 \text{ i} \text{ 0}^8 \text{ u1}^{16} \text{ u2}^2 + 369098752 \text{ i} \text{ 0}^{10} \text{ u1}^{16} \text{ u2}^2 - 973078528 \text{ i} \text{ 0}^{10} \text{ u2}^2 + 369098752 \text{ i} \text{ 0}^{10} \text{ u1}^{16} \text{ u2}^2 + 369098752 \text{ i} \text{ 0}^{10} \text{ u1}^{16} \text{ u2}^2 + 369098752 \text{ i} \text{ 0}^{10} \text{ u1}^{16} \text{ u2}^2 + 369098752 \text{ i} \text{ 0}^{10} \text{ u1}^{16} \text{ u2}^2 + 369098752 \text{ i} \text{ 0}^{10} \text{ u1}^{10} \text{ u2}^2 + 369098752 \text{ i} \text{ 0}^{10} \text{ u1}^{10} \text{ u2}^2 + 369098752 \text{ i} \text{ 0}^{10} \text{ u1}^{10} \text{ u2}^2 + 369098752 \text{ i} \text{ 0}^{10} \text{ u1}^{10} \text{ u2}^2 + 369098752 \text{ i} \text{ 0}^{10} \text{ u1}^{10} \text{ u2}^2 + 369098752 \text{ i} \text{ 0}^{10} \text{ u1}^{10} \text{ u2}^{10} \text{ u1}^{10} \text{ u2}^{10} \text{ u1}^{10} \text{ u2}^{10} \text{ u1}^{10} \text{ u1}^{10} \text{ u1}^{10} \text{ u2}^{10} \text{ u1}^{10} \text{ u1}^{10} \text{ u2}^{10} \text{ u1}^{10} \text{ u1}^{10} \text{ u2}^{10} \text{ u1}^{10} \text{ u2}^{10} \text{ u1}^{$

 $30298210304 Q^9 u1^{10} u2 + 39352008704 Q^{11} u1^{10} u2 - 24798822400 Q^{13} u1^{10} u2 + 7532969984 Q^{15} u1^{10} u2 - 939524096 Q^{17} u1^{10} u2 + 44515328 i Q^2 u1^{11} u2 - 840368128 i Q^4 u1^{11} u2 + 6171262976 i Q^6 u1^{11} u2 - 21483225088 i Q^8 u1^{11} u2 + 35381051392 i Q^{10} u1^{11} u2 - 21483225088 i Q^8 u1^{11} u2 + 35381051392 i Q^{10} u1^{11} u2 - 21483225088 i Q^8 u1^{11} u2 + 35381051392 i Q^{10} u1^{11} u2 - 21483225088 i Q^8 u1^{11} u2 + 35381051392 i Q^{10} u1^{11} u2 - 21483225088 i Q^8 u1^{11} u2 + 35381051392 i Q^{10} u1^{11} u2 - 21483225088 i Q^8 u1^{11} u2 + 35381051392 i Q^{10} u1^{11} u2 - 21483225088 i Q^8 u1^{11} u2 + 35381051392 i Q^{10} u1^{11} u2 - 21483225088 i Q^8 u1^{11} u2 + 35381051392 i Q^{10} u1^{11} u2 - 21483225088 i Q^8 u1^{11} u2 + 35381051392 i Q^{10} u1^{11} u2 - 21483225088 i Q^8 u1^{11} u2 + 35381051392 i Q^{10} u1^{11} u2 - 21483225088 i Q^8 u1^{11} u2 + 35381051392 i Q^{10} u1^{11} u2 - 21483225088 i Q^8 u1^{11} u2 + 35381051392 i Q^{10} u1^{11} u2 - 21483225088 i Q^8 u1^{11} u2 + 35381051392 i Q^{10} u1^{11} u2 - 21483225088 i Q^8 u1^{11} u2 + 35381051392 i Q^{10} u1^{11} u2 - 21483225088 i Q^8 u1^{11} u2 + 35381051392 i Q^{10} u1^{11} u2 - 21483225088 i Q^8 u1^{11} u2 + 35381051392 i Q^{10} u1^{11} u2 - 21483225088 i Q^8 u1^{11} u2 + 35381051392 i Q^{10} u1^{11} u2 - 21483225088 i Q^8 u1^{11} u2 + 35381051392 i Q^{10} u1^{11} u2 - 21483225088 i Q^8 u1^{11} u2 + 35381051392 i Q^{10} u1^{11} u2 - 35881051392 i Q^{10} u1^{11} u2 + 35881051392 i Q^{10} u1^{11} u2 + 35881051392 i Q^{10} u1^{11} u2 + 35881051392 i Q^{10} u1^{11} u2 - 35881051392 i Q^{10} u1^{11} u2 + 358810510 u1^{11} u2 + 358810510 u1^{11} u2 + 358810510 u1$

- But the sum can be done by maple
- At four loop it match with Feynman diagrams

- No five loop yet
- But S-matrix gives EXACT formula!

• Exact solution



Conclusion



perturbation

Non-perturbation

Rigorous, analytic understanding based on integrability is at the core of non-perturbative phenomena



Thank you