섭동의 한계를 넘어서

가적분성 Integrability
toward Non-perturbative physics

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“Non” in Physics

Non-linear

Non-Fermi

Non-Gaussian

$\boxed{\text{Non-perturbative}}$

…

섭동 [perturbation , 攝動]

- 고전역학이나 양자역학계의 운동을 기술하는 해밀턴함수 또는 연산자가 비교적 간단한 주부(主部) 외에 작은 부가 항을 포함하는 경우 이것을 전자(前者)가 정하는 운동에 작은 흩어짐을 가하는 것으로 간주하여 섭동이라고 하고, 이것을 다루는 이론을 섭동론이라 한다. 이 이론은 작은 원인으로부터 작은 결과가 예상되는 모든 계통에 적용되므로 방정식 자체의 섭동 외에 경제에 있어서의 그것까지 포함하여 꽤 넓은 범위에 걸쳐서 근사적(近似的)인 주요 수단을 주고 있다. – 네이버 지식사전

(예) 양자역학

- 섭동에 의한 에너지준위의 변화

$$H|\Psi_n\rangle = E_n|\Psi_n\rangle$$

$$H = H_0 + \lambda H', \qquad H_0|\Psi_n^0\rangle = E_n^0|\Psi_n^0\rangle$$

$$E_n = E_n^0 + \lambda\langle\Psi_n^0|H'|\Psi_n^0\rangle + \lambda^2 \sum_{m\neq n} \frac{|\langle\Psi_n^0|H'|\Psi_m^0\rangle|^2}{E_n^0 - E_m^0} + \cdots$$

- 변수λ가 충분히 작을 때만 유효함

Taylor expansion

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + \ldots$$

Valid if $x \ll 1$

Invalid if $x > 1$

Need many terms if $x < 1$

섭동이론의 성공

- 섭동이론은 매우 많은 현상들에 적용가능

- 20세기 후반의 이론물리학은 대부분 섭동이론에 기반
- Feynman diagram

$$E_n = E_n^0 + \lambda \langle \Psi_n^0 | H' | \Psi_n^0 \rangle + \lambda^2 \sum_{m \neq n} \frac{|\langle \Psi_n^0 | H' | \Psi_n^0 \rangle|^2}{E_n^0 - E_m^0} + \dots$$


- (예) 전자기 상호작용

전자의 자기모멘트

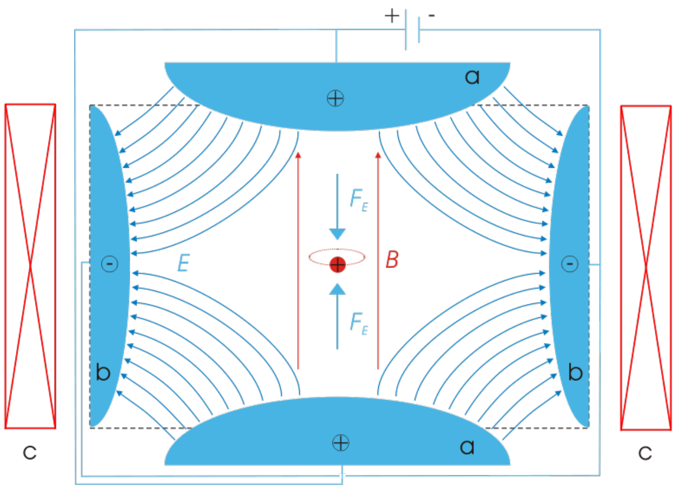
- g-factor

$$\boldsymbol{\mu}_e = g \frac{(-e)}{2m_e} \boldsymbol{S}$$

  – Dirac 방정식 $g = 2$

  – 실험: $a_e \equiv \frac{g-2}{2} = 0.001\ 159\ 652\ 180\ 85(76)$

Penning Trap



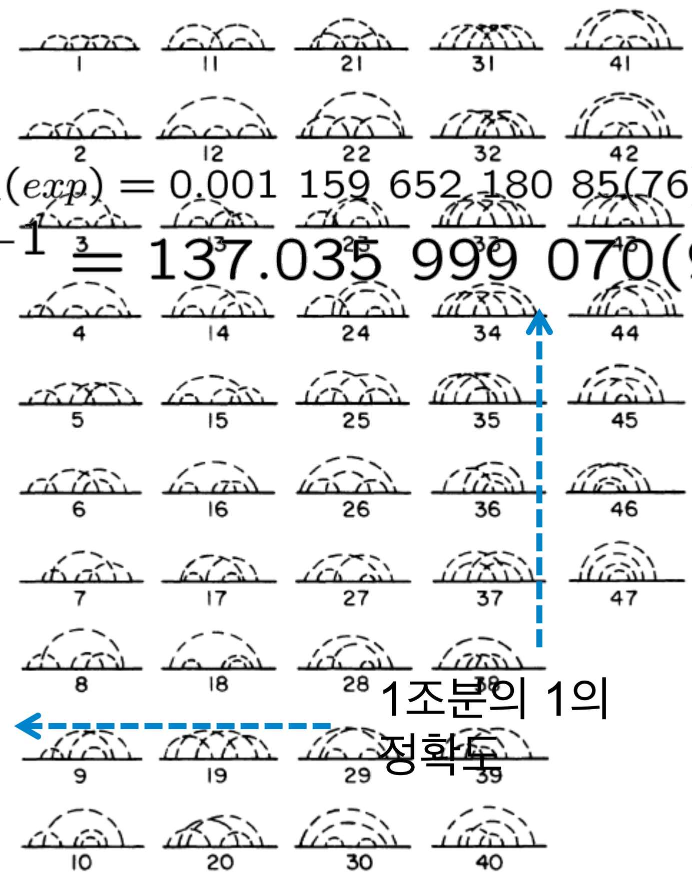
양자동역학 (Quantum Electrodynamics)

- QED: 양자역학+전자기학 (Feynman,…)

$$
\begin{aligned}
a_e &= 0.5\left(\frac{\alpha}{\pi}\right) \\
&- 0.328\ 478\ 965\ 579\left(\frac{\alpha}{\pi}\right)^2 \\
&+ 1.181\ 241\ 456\ 587\left(\frac{\alpha}{\pi}\right)^3 \\
&- 1.509\ 8(384)\left(\frac{\alpha}{\pi}\right)^4
\end{aligned}
$$

$$a_e(exp) = 0.001\ 159\ 652\ 180\ 85(76)$$

$$\rightarrow \alpha^{-1} = 137.035\ 999\ 070(98)$$



1조분의 1의 정확도

$$\alpha \equiv \frac{e^2}{4\pi\varepsilon_0\hbar c} = \frac{1}{137.035\ 999\ 074(44)}$$

Measured by Qauntum Hall Effect

인류가 만든 이론들 중 실험과 가장 정밀하게 확인된 이론

Fundamental Forces in Nature

- 전자기력
  - 응집물리계
  - 양자전기동역학(QED)
- 약력
  - 입자물리학의 표준모형
  - 전자기력 정도로 상호작용이 작음
- 중력
  - 전자기력보다 더 약함
- 강력

강력 (QCD)

- 양성자안 쿼크들간의 상호작용은 매우 약함
  - Asymptotic freedom (2004년 Nobel 물리학상)

거리가 매우 가깝거나 에너지가 매우 클 경우

$$g_s \rightarrow 0$$

  - 강력도 섭동으로 설명 가능

- 쿼크들 사이가 멀어지면 상호작용이 강해져서 섭동계산 불가능

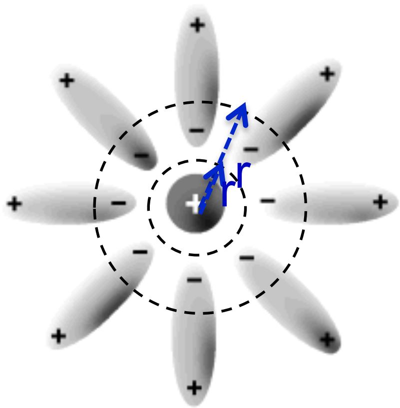
섭동이론으로 충분한가?

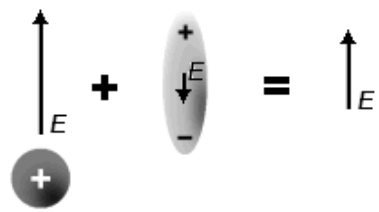
No!

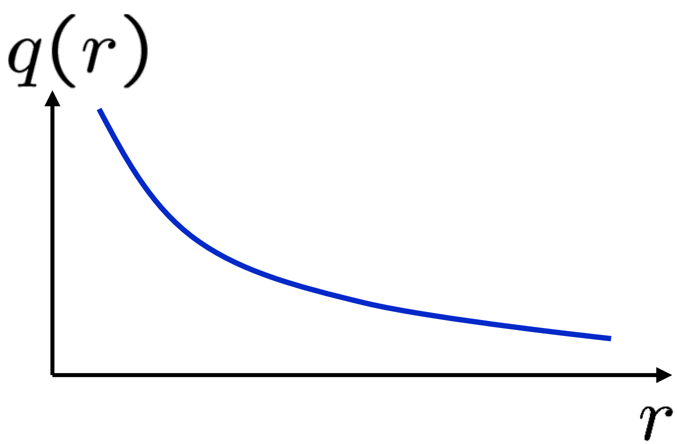
섭동이론이 적용되지 않는

많은 현상들이 존재

재규격화 (Renormalization)

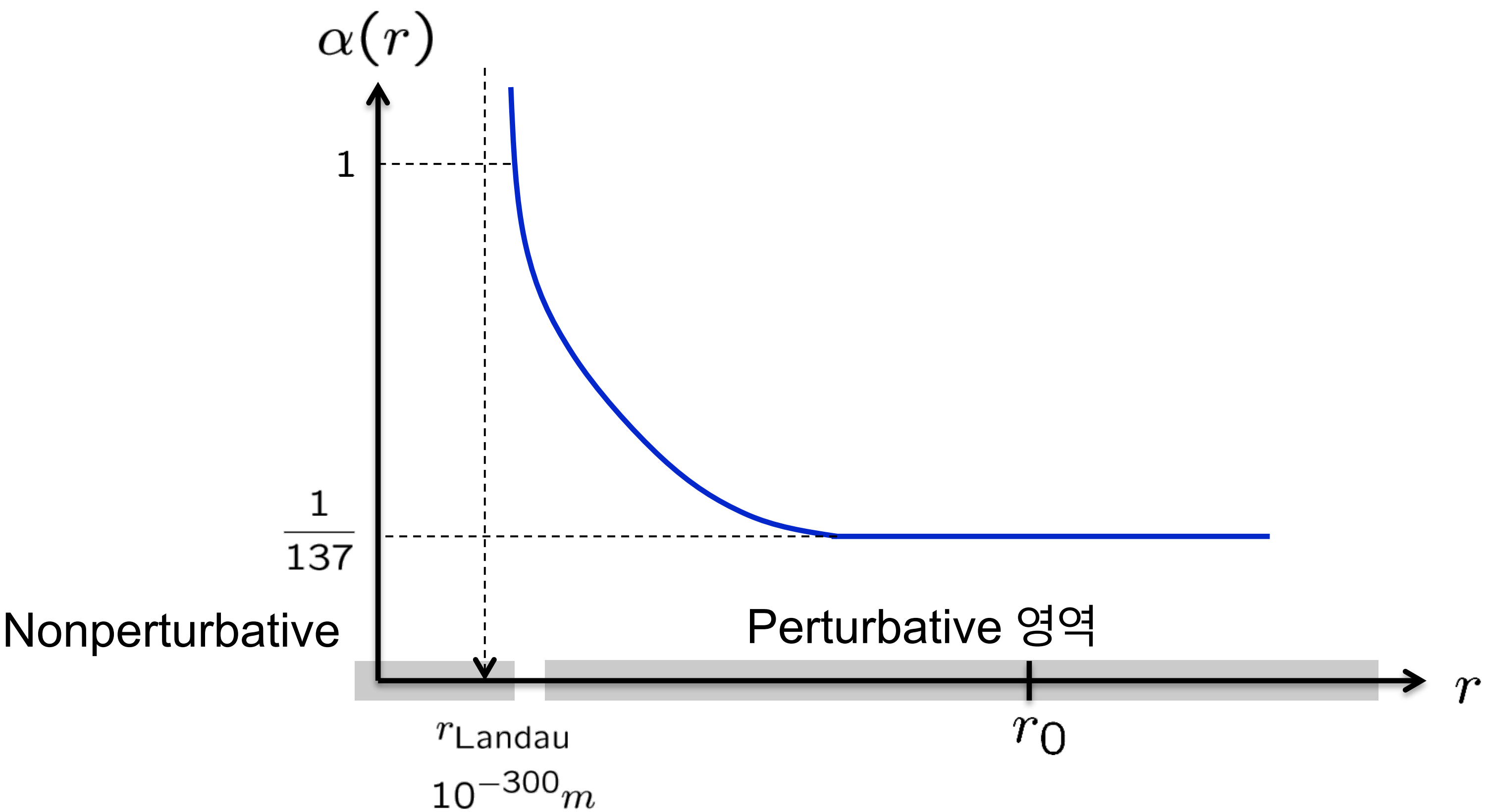






전자기력(QED)

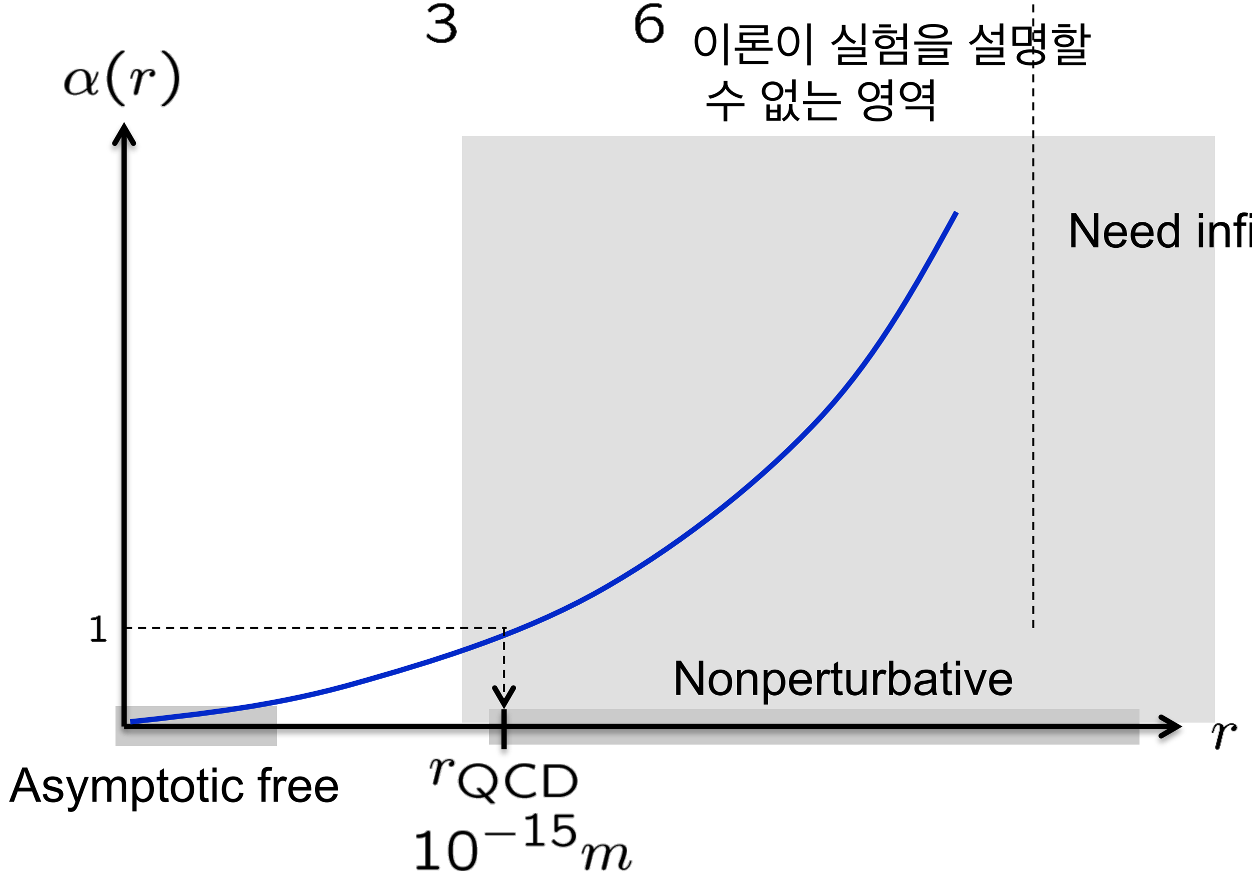
$$\alpha(r) = \alpha - \frac{\alpha^2}{3\pi}\log\frac{r}{r_0} + \ldots \approx \frac{\alpha(r_0)}{1 + \frac{\alpha(r_0)}{3\pi}\log\frac{r}{r_0}}$$



강력(QCD)

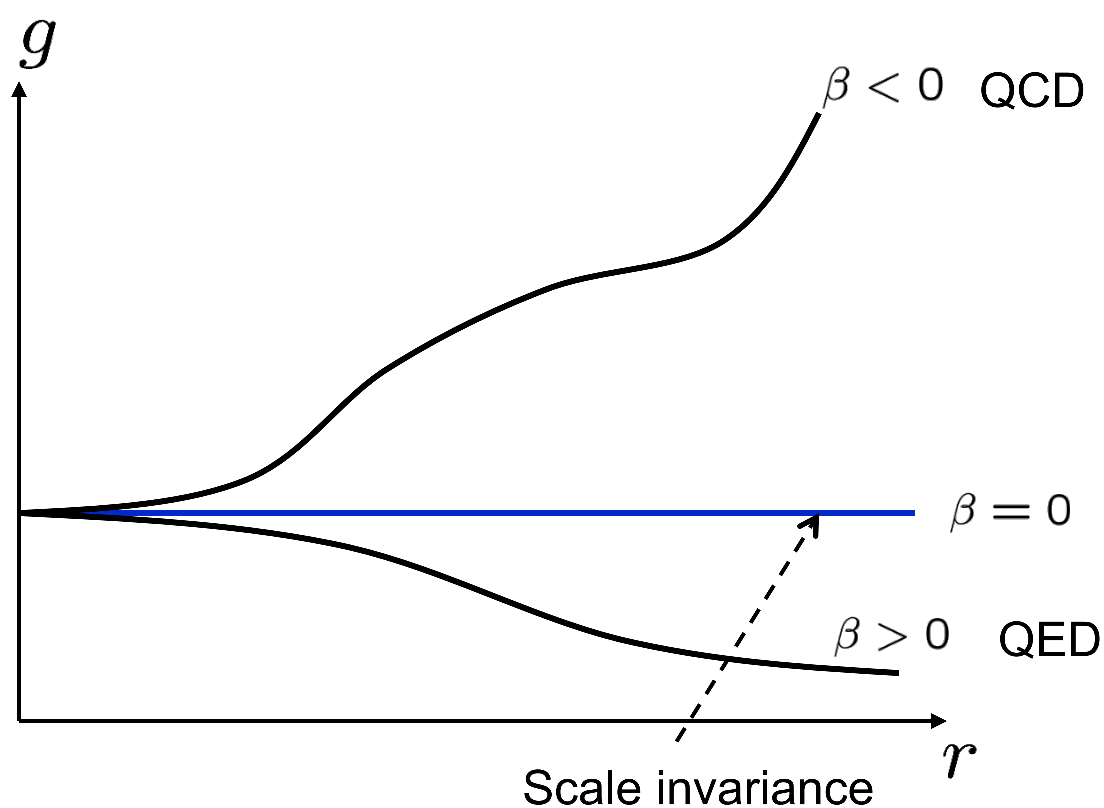
$$\alpha_s(r) = \alpha_s + \alpha_s^2\left(\frac{11}{3}N_c - \frac{2}{3}n_f\right)\log\frac{r}{r_0} + \dots \qquad \approx \cancel{\frac{\alpha_s(r_0)}{1-7\alpha_s(r_0)\log\frac{r}{r_0}}}$$

3 6



재규격화군(Renormalization group)

$$\frac{d}{d \log r} g \equiv -\beta(g)$$



응집물질계의 전자기력

Fermi liquid

강상관계

Strongly correlated electron system

Non-Fermi liquid

비섭동이론

- 매우 난해한 문제
- 유일한 방법은 EXACT SOLUTION
- 가적분성 (INTEGRABILITY)

SYMMETRY → exact solution

가적분 모형

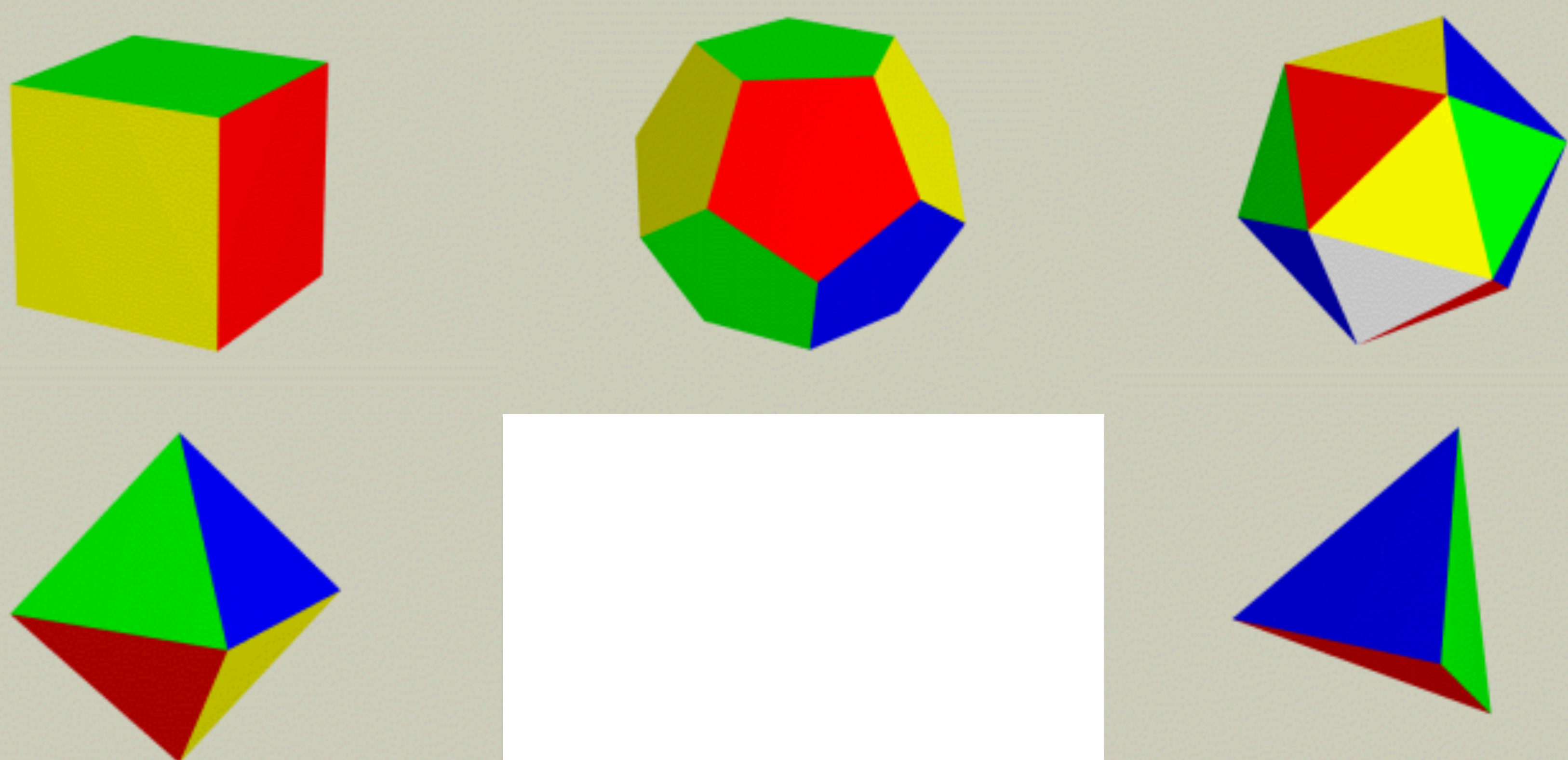
- 무한개의 보존전하들을 갖는 특별한 이론모형
  - (cf) 수소원자: 3차원 회전대칭성 su(2)

$$\{H,\ L^2,\ L_z,\ S^2,\ S_z\}$$

- 다양한 수학적 대칭성
- 기술의 발전으로 실험에서 입증가능
- 2차원 시공간에서 정의되지만 4차원 시공간의 물리현상에 적용가능

대칭성 Symmetry

- 물리학자들이 자연을 보는 믿음
- (예) 플라톤의 완전도형



두가지 비섭동/대칭성의 예들

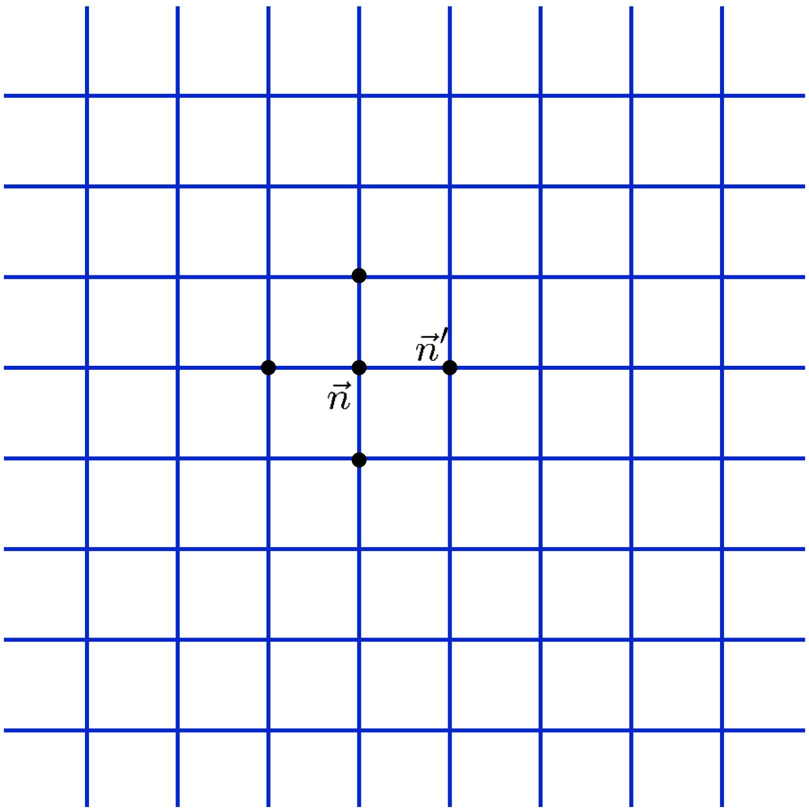
1. 저차원 응집물리계

2. 4차원 게이지이론 / 10차원 초끈

2D Ising model

$$E = -J \sum_{(\vec{n},\vec{n}')} s_{\vec{n}} s_{\vec{n}'} + B \sum_{\vec{n}} s_{\vec{n}}$$

$s_{\vec{n}} = \pm 1$



Partition function

$$Z = \sum_{\{s\}} e^{-E/k_B T}$$

임계점

$$\frac{J}{k_B T_c} = \frac{1}{2}\text{asinh}(1) = 0.4406\ldots$$

$$B_c = 0$$

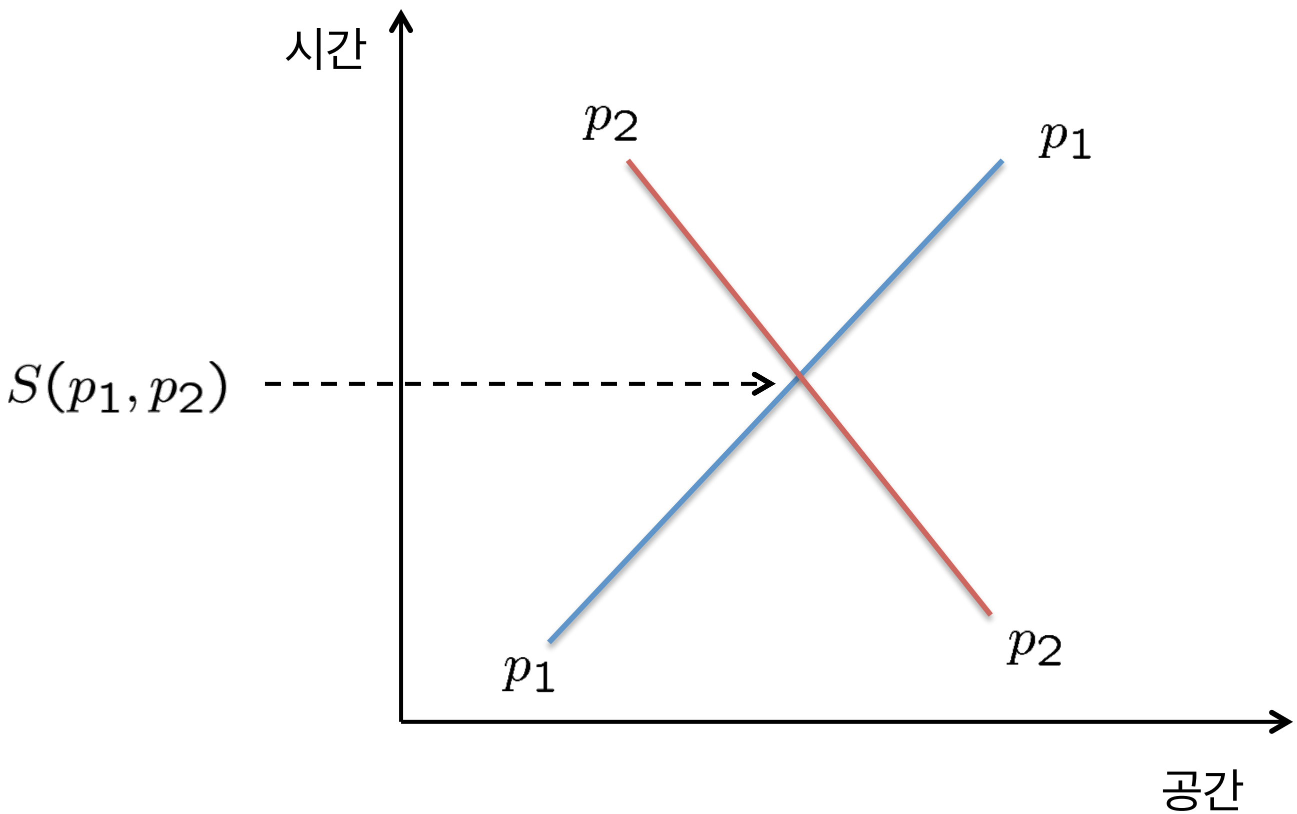
Onsager (1944)

$$B = 0$$

Can we solve for $B \neq 0$?

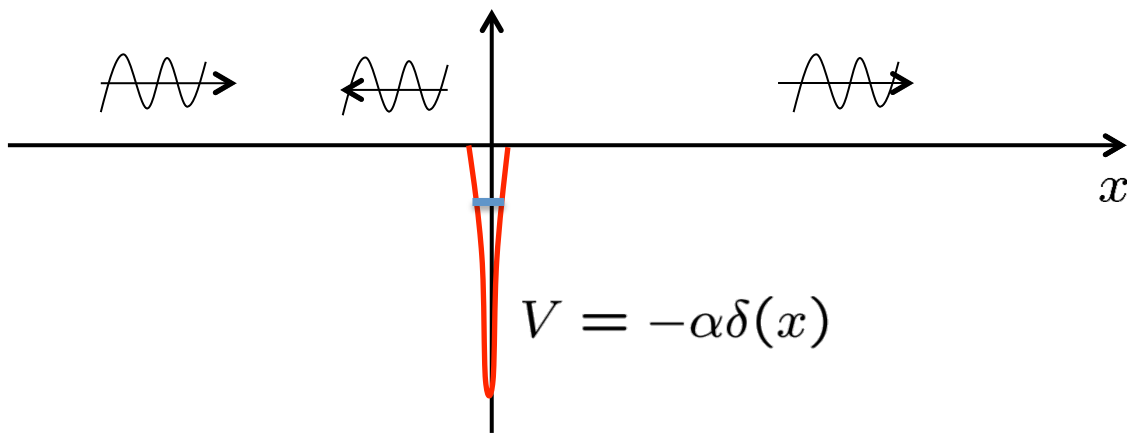
S-행렬

- 무한개의 보존전하 → 완전탄성산란행렬



- S-행렬로부터 정확한 물리량 계산가능

- Pole of S → bound state [구속입자]
  - (ex) 양자역학의 delta함수 퍼텐샬



  - 구속 energy:

$$E = -\frac{m\alpha^2}{2\hbar^2}$$

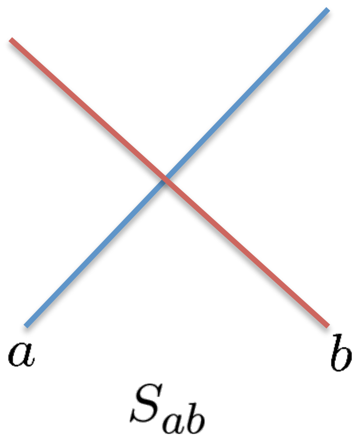
Griffiths pp.73-74

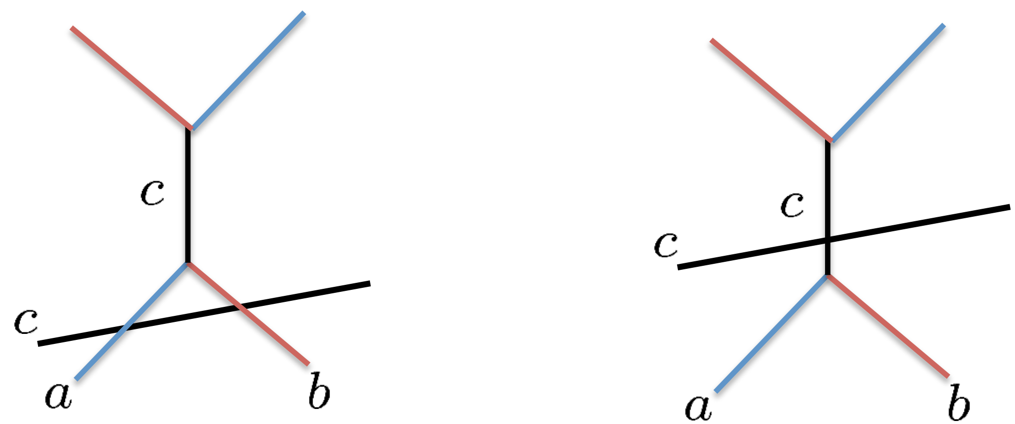
  - Scattering amplitudes: (E>0)

$$r(p) = \frac{i\beta p}{p - i\beta}, \qquad t(p) = \frac{i\beta p}{p - i\beta}, \qquad \beta \equiv \frac{m\alpha}{\hbar}$$

Pole at $p = i\beta$ -----> $E = -\dfrac{m\alpha^2}{2\hbar^2}$

Bootstrap





$$S_{ca}S_{cb} = S_{cc}$$

2D Ising 모형의 비섭동적 결과

- S-matrices by Zamolodchikov (1988)

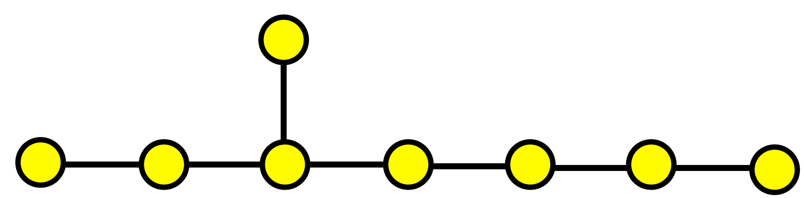
- Mass spectrum from poles of S-matrices

$$
\begin{aligned}
m_1 &= (4.40490857\ldots)|B|^{5/13}, \\
m_2 &= 2m_1 \cos\frac{\pi}{5}, \dashrightarrow \frac{\sqrt{5}+1}{2} \\
m_3 &= 2m_1 \cos\frac{\pi}{30}, \\
m_4 &= 4m_1 \cos\frac{\pi}{5}\cos\frac{7\pi}{30}, \\
m_5 &= 4m_1 \cos\frac{\pi}{5}\cos\frac{2\pi}{15}, \\
m_6 &= 4m_1 \cos\frac{\pi}{5}\cos\frac{\pi}{30}, \\
m_7 &= 8m_1 \cos\frac{\pi}{5}\cos\frac{\pi}{5}\cos\frac{7\pi}{30}, \\
m_8 &= 8m_1 \cos\frac{\pi}{5}\cos\frac{\pi}{5}\cos\frac{2\pi}{15}
\end{aligned}
$$

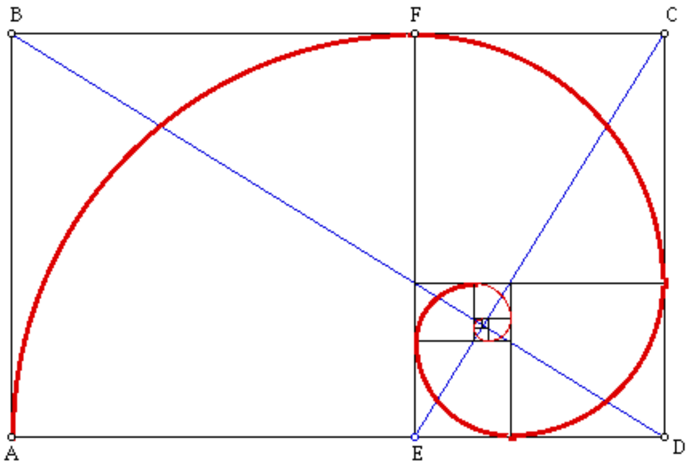
황금비

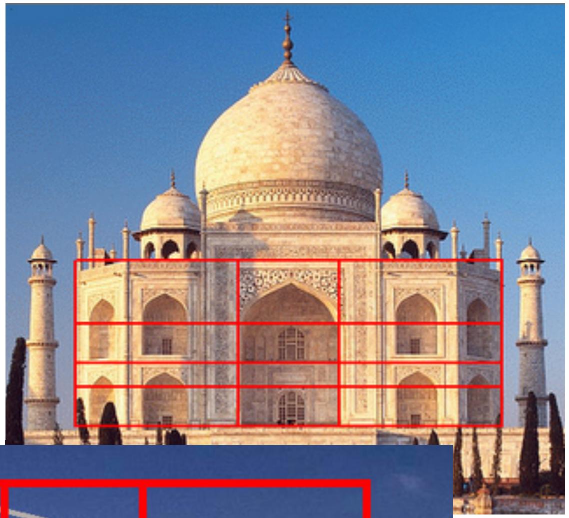
Perron-Frobenius vector
of Cartan matrix

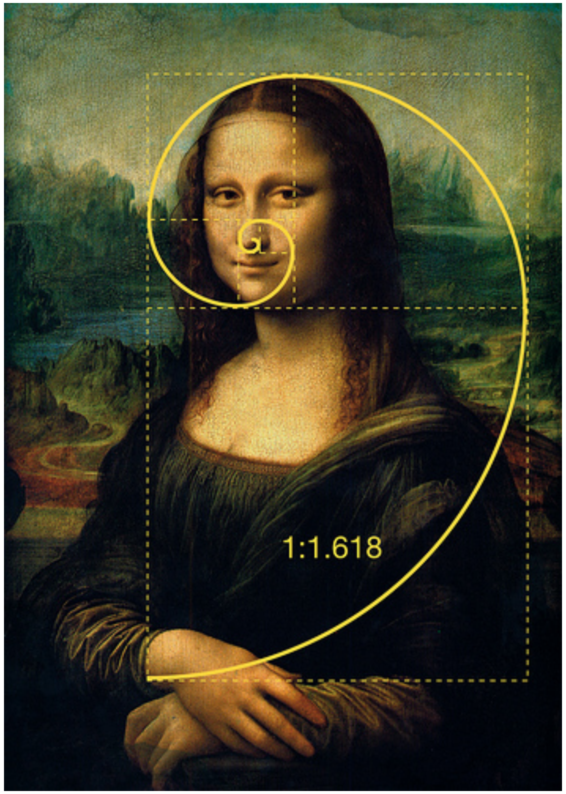
E_8 Lie algebra

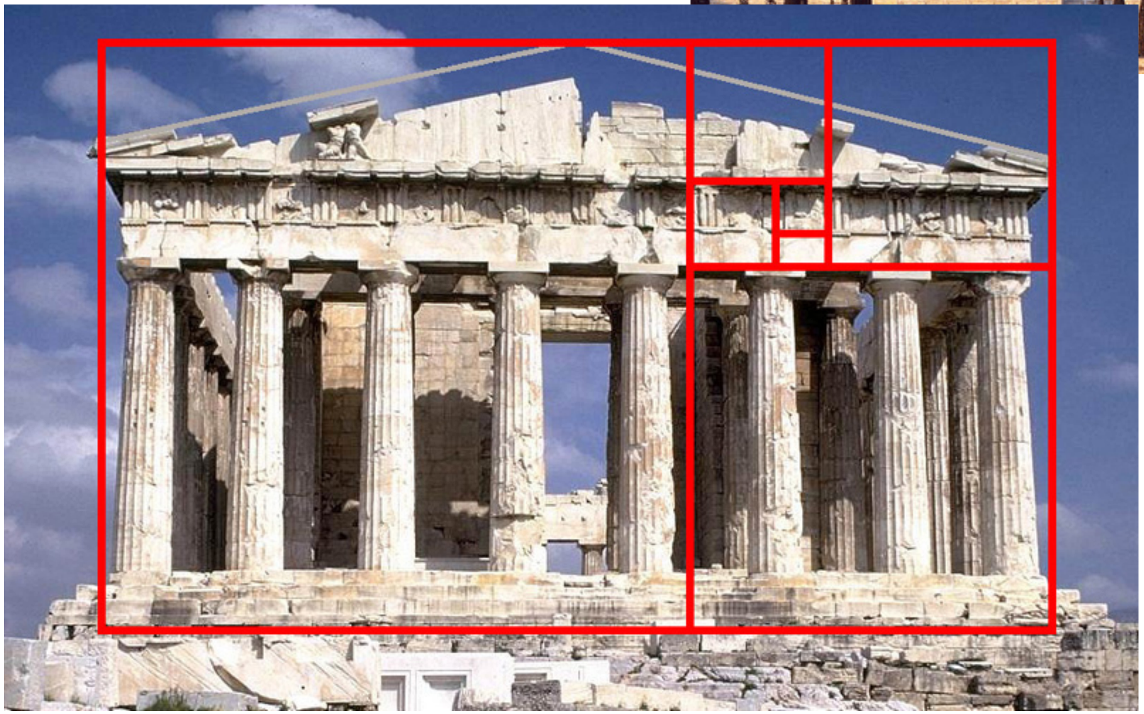


황금비 Golden ratio



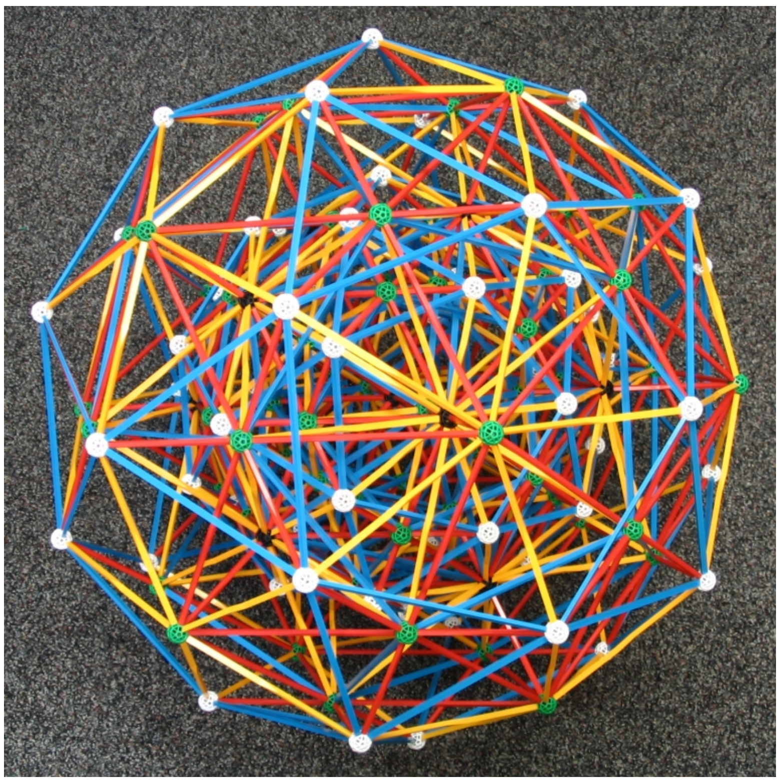


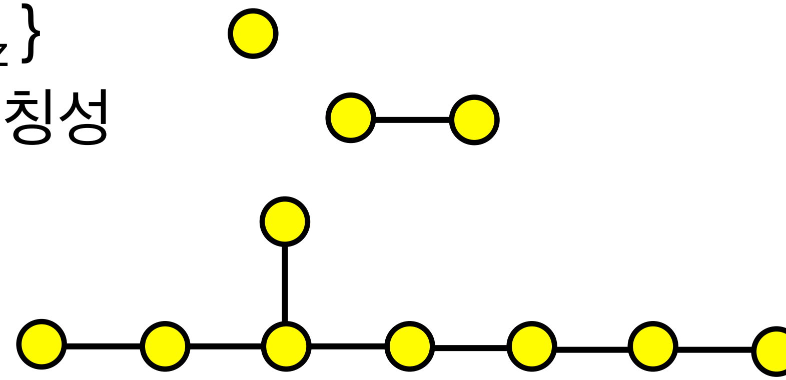




E_8 symmetry

- Highest-order symmetry in mathematics
  - su(2): 3차원 회전대칭성 $\{L_x, L_y, L_z\}$
  - su(3): 8차원 회전대칭성, 강력의 대칭성
  - E_8 : 248차원 회전대칭성, 초끈이론

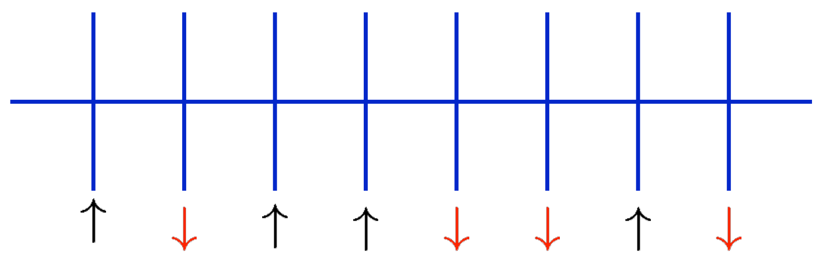




실험실에서 구현된 E_8

- 2D Ising model = 1D Ising spin chain

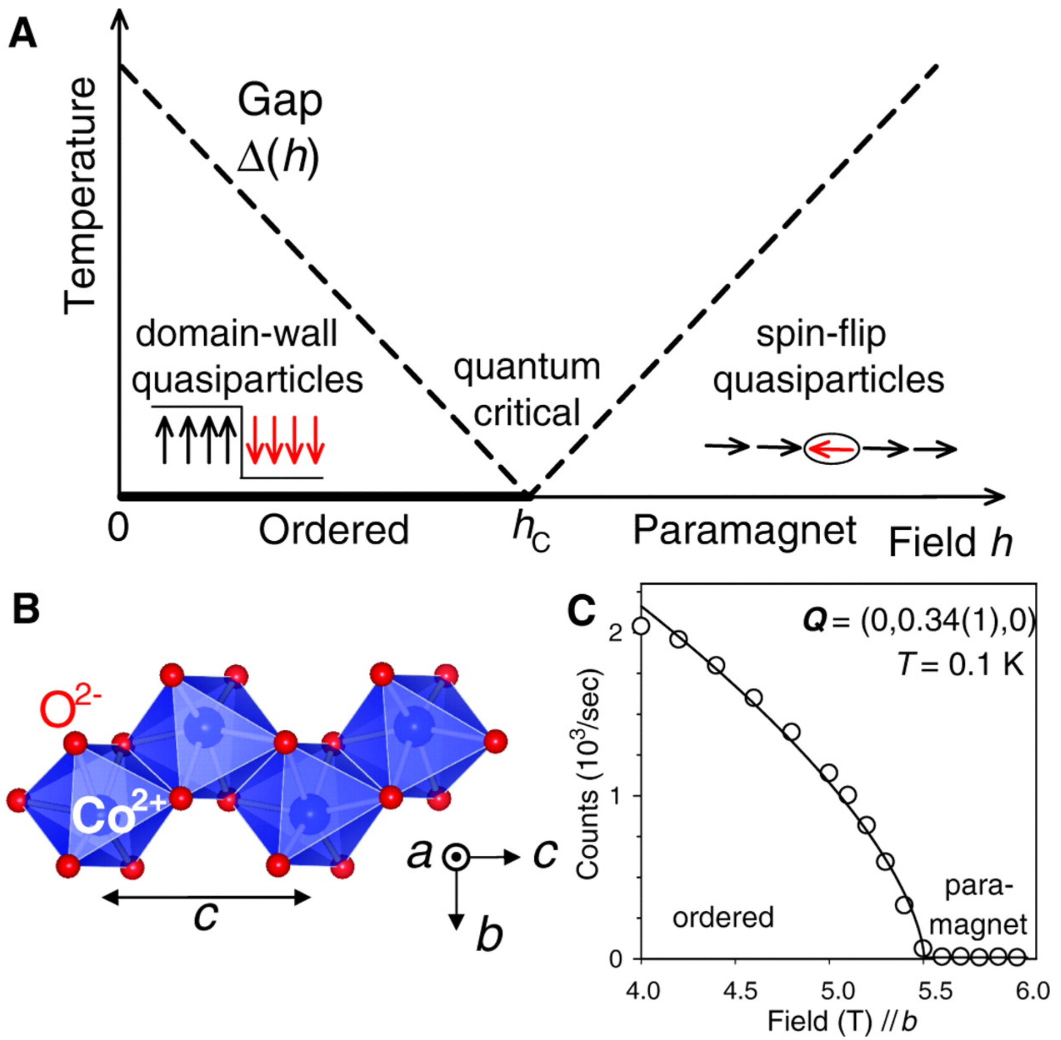
$$\hat{H} = -J \sum_{j=1} \left[\sigma_j^z \sigma_{j+1}^z + h\sigma_j^x\right] + B\sigma_j^z$$



$$h = h_c = \frac{1}{2}$$ Critical Ising model

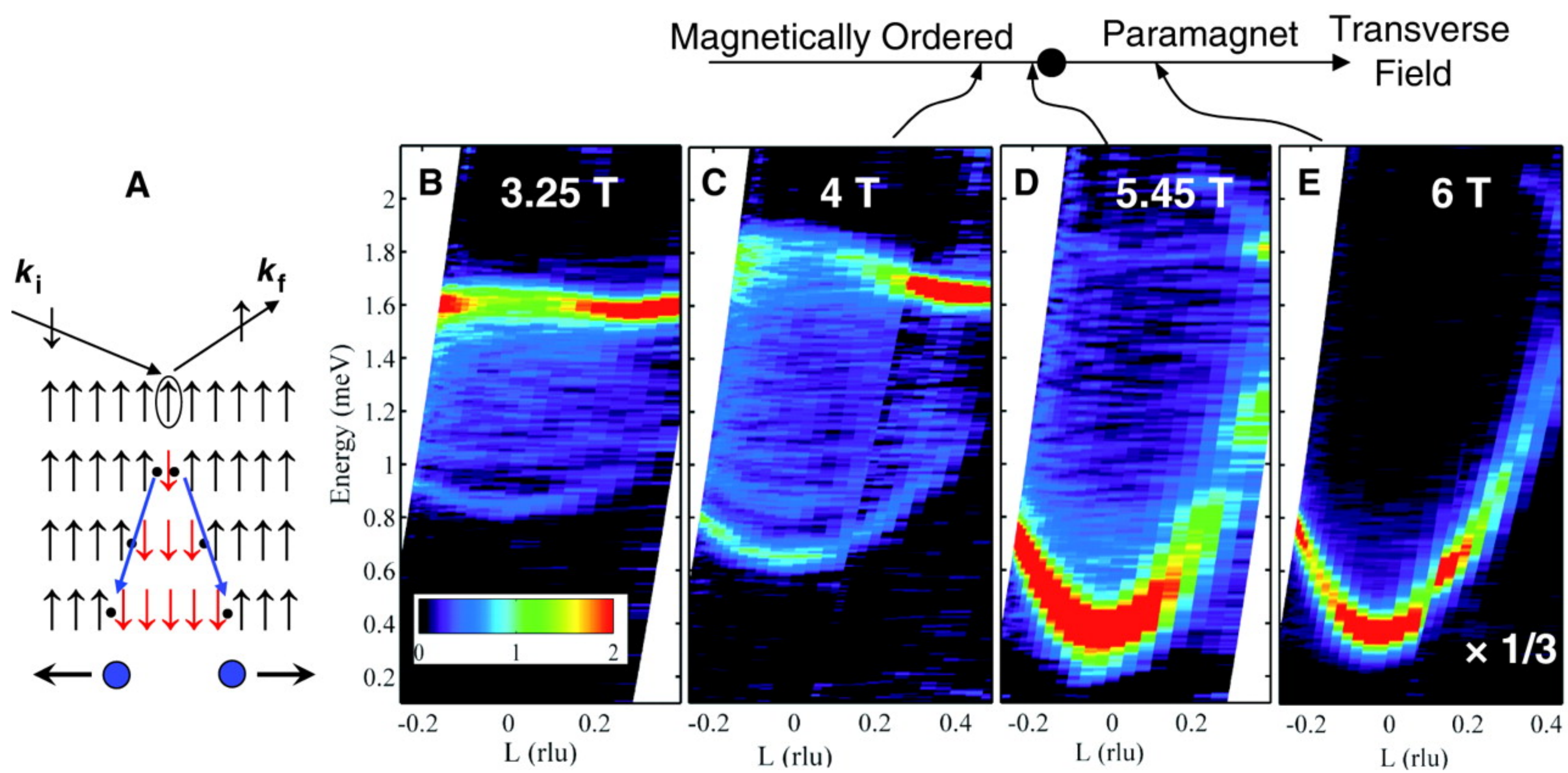
- Material: $CoNb_2O_6$

B=0



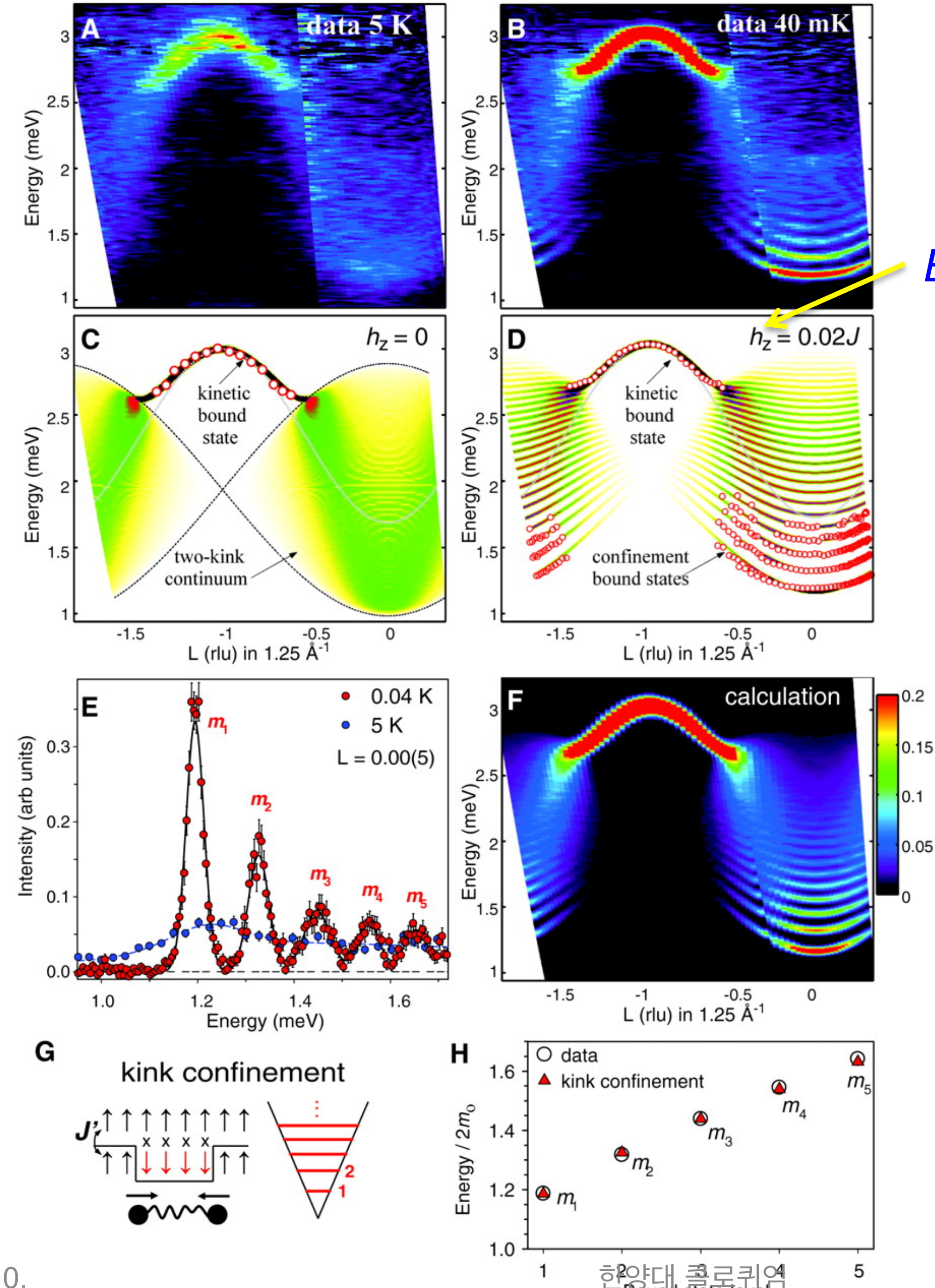
R Coldea et al. Science 2010;327:177-180

(A) Cartoon of a neutron spin-flip scattering that creates a pair of independently propagating kinks in a ferromagnetically ordered chain.



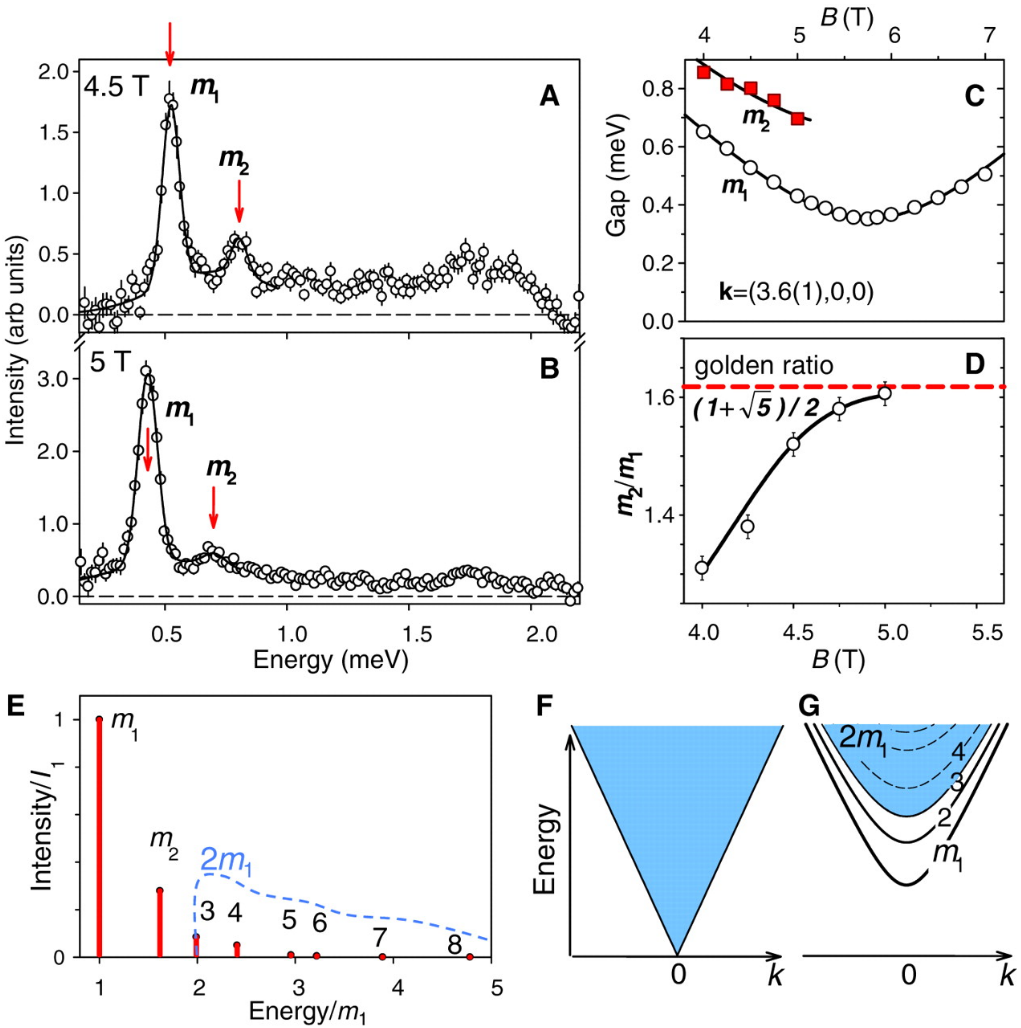
R Coldea et al. Science 2010;327:177-180

Zero-field spin excitations in CoNb2O6.



B

(A and B) Energy scans at the zone center at 4.5 and 5 T observing two peaks, m1 and m2, at l ow energies.

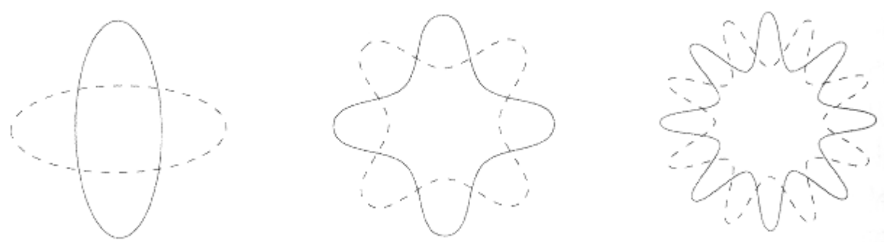


R Coldea et al. Science 2010;327:177-180

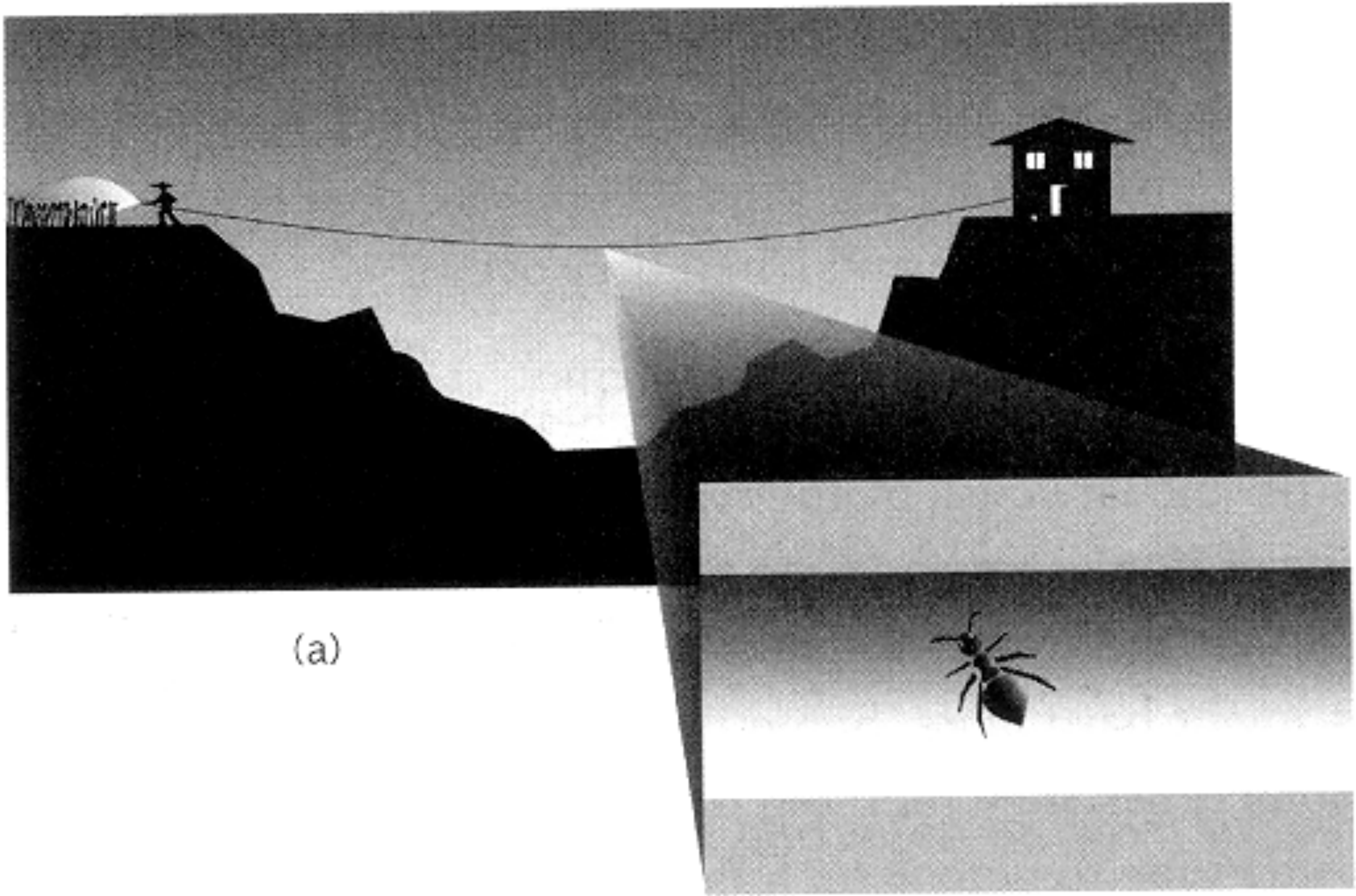
중력 / 강력 이중성 (duality)

양자중력

- Planck scale $10^{-35}m$
- 중력의 양자화 필요
- 초끈이론: Planck scale의 1차원 물체



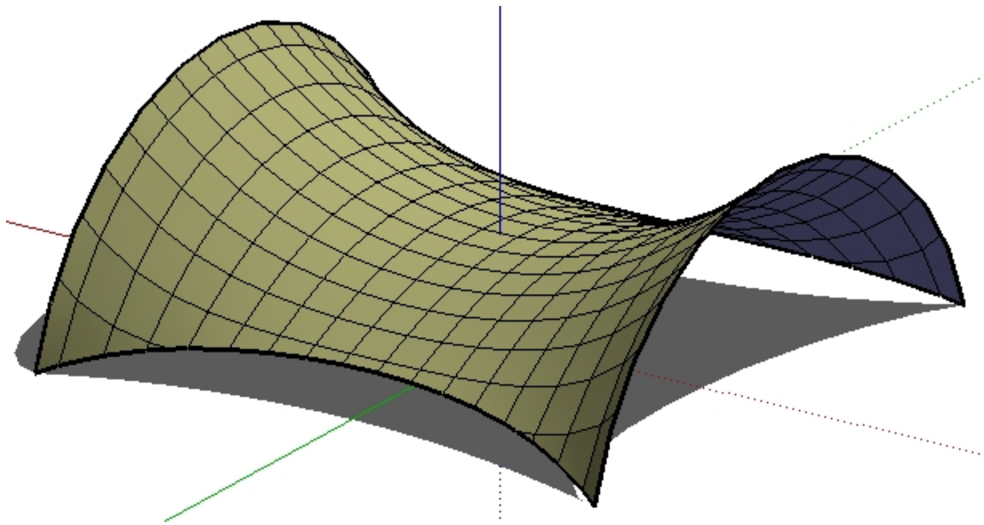
- 수학적으로 10차원에서만 가능
  - 4차원시공간을 제외하면 매우 작은 크기



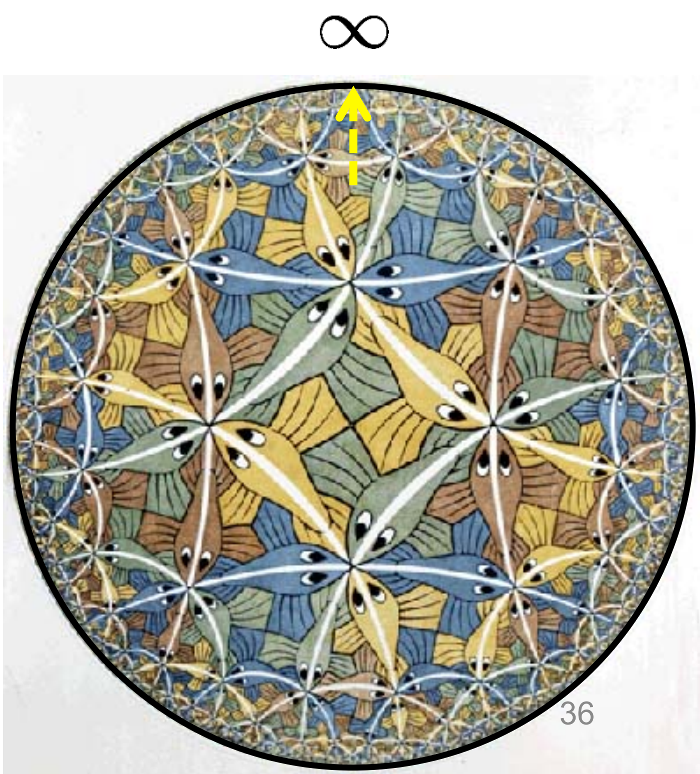
(a)

(b)

음의 곡률을 가진 공간

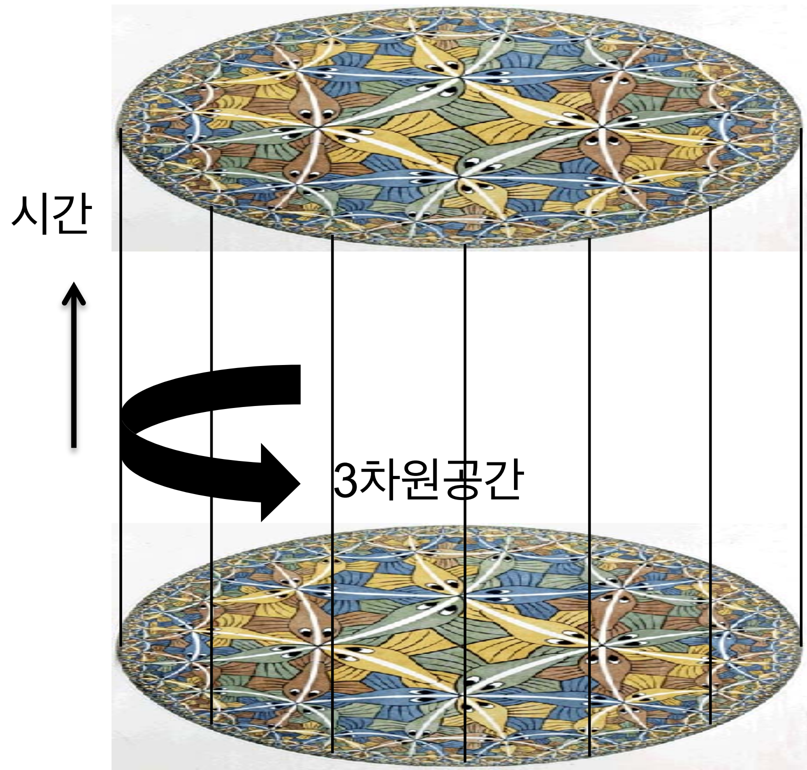


- Anti-de Sitter (AdS) 공간



AdS 공간

- 내부: 5차원 AdS_5 공간



- 경계: 4차원 시공간

Holography 원리

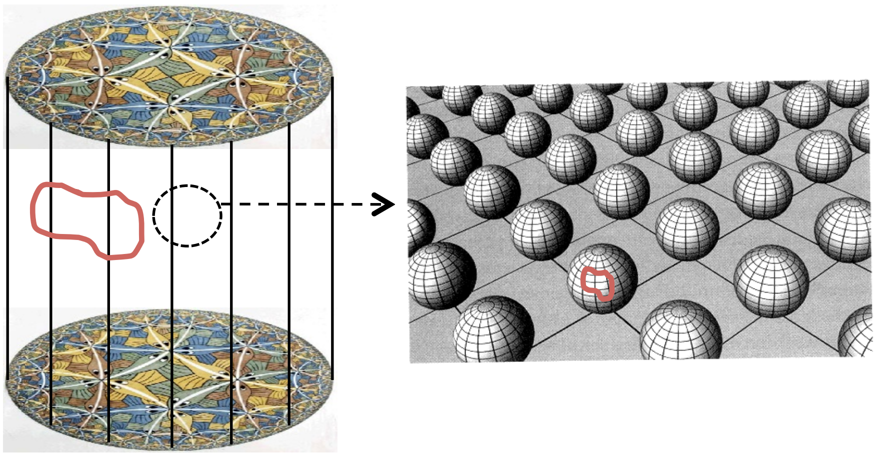
- 홀로그램

- Holography 원리
  - AdS공간의 초끈이론 = AdS경계공간의 게이지이론
  - 양자중력효과 ←→ 상호작용의 크기

$$\frac{1}{\lambda} \qquad\qquad\qquad \lambda$$

AdS/CFT 가설 (Maldacena)

- AdS_5 x S^5 공간에서 움직이는 초끈이론



- 4차원 초대칭적 양-밀스 이론

게이지이론

- 전자기 (Abelian gauge theory)

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} = \frac{1}{2}\left(\mathbf{E}^2 - \mathbf{B}^2\right) \qquad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

- 양-밀스 이론: $A_\mu = N_c \times N_c$ Matrix

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + ig[A_\mu, A_\nu]$$

$$\mathcal{L} = -\frac{1}{4}\mathrm{Tr}\left[F_{\mu\nu}F^{\mu\nu}\right] = \frac{1}{2}\sum_a \left(\mathbf{E}_a{}^2 - \mathbf{B}_a{}^2\right)$$

초대칭적 양-밀스 이론

- Supersymmetry (초대칭성)
  - gauge boson, fermion, scalar boson

    Spin 1 → Spin 1/2 → Spin 0

  - N=4 SYM $(A_\mu, \chi^a_\alpha, \Phi^j), \quad a = 1, \ldots, 4; j = 1, \ldots, 6$

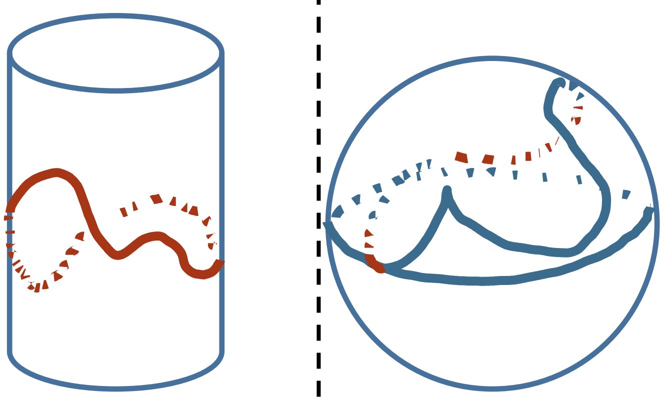
$$S = \frac{\mathrm{Tr}}{g^2_{\mathrm{YM}}} \int d^4x \left\{ -\frac{1}{4} F^2_{\mu\nu} + (D_\mu \Phi^a)^2 + \left[\Phi^a, \Phi^b\right]^2 + \bar{\chi} \not{D} \chi - i \bar{\chi} \Gamma_a [\Phi^a, \chi] \right\}$$

  - scale invariance (conformal field theory: CFT)

$$\beta \equiv \mu \frac{\partial g_{\mathrm{YM}}}{\partial \mu} = -\frac{g^3_{\mathrm{YM}}}{16\pi^2} \left(\frac{11}{3} N_c - \frac{1}{6} \overbrace{\sum_i C_i}^{6N_c} - \frac{1}{3} \overbrace{\sum_j \tilde{C}_j}^{8N_c} \right) = 0$$

AdS/CFT 이중성

Energy of string = Dimension of SYM



$$\mathcal{O}(x) = \mathrm{Tr}\left[XYZF_{\mu\nu}\chi^{\alpha}(D_{\mu}Y)\ldots\right]$$

$$\langle\mathcal{O}(x)\mathcal{O}(0)\rangle = \frac{1}{|x|^{2\Delta}}$$

Large λ일 때만 계산가능

비섭동적 결과 필요

$$\lambda = g_{\mathrm{YM}}^2 N_c$$

Symmetry

↓

S-matrix

↓

비섭동적 결과

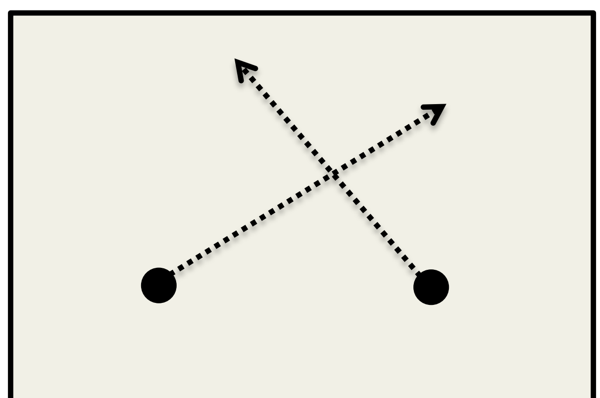
S-matrix

- SYM : scattering of fields on the spin chain

$$\mathrm{Tr}\,[\cdots ZZZZZZZZZ\overset{\overrightarrow{\;p_1\;}}{\chi_1}ZZZZZZ\cdots ZZZZZZ\overset{\overleftarrow{\;p_2\;}}{\chi_2}ZZZZZZZZZZ\cdots]$$

$$\left\{X,\bar{X},Y,\bar{Y},\chi^{\alpha},D_{\mu}\right\}$$

- String side : scattering on the world sheet

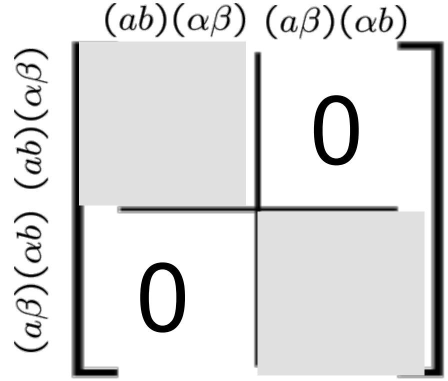


- Symmetry : 4d conformal group psu(2,2|4)

- From symmetry to S-matrix

$$\left[\mathbf{S}(p_1,p_2),\left(\begin{array}{c|c}\mathbb{L}_a{}^b & \mathbb{Q}_\alpha{}^b \\ \hline \mathbb{Q}_a^{\dagger\beta} & \mathbb{R}_\alpha{}^\beta\end{array}\right)\right]=0$$

- S : 16 x 16 matrix

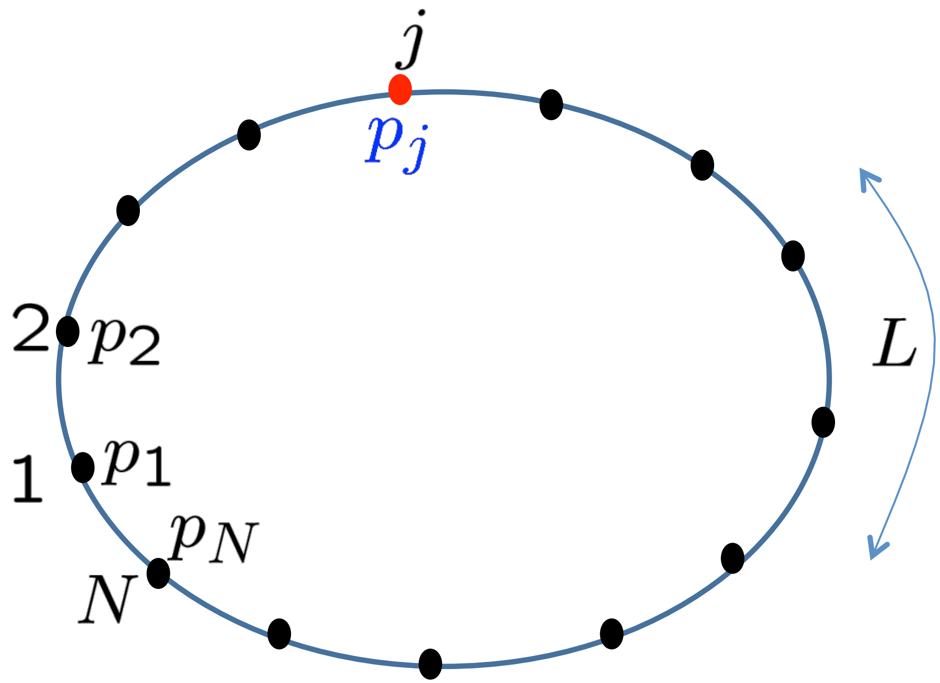


$$
\begin{aligned}
S^{a\,a}_{a\,a} &= A\,, \quad S^{\alpha\,\alpha}_{\alpha\,\alpha} = D\,,\\
S^{a\,b}_{a\,b} &= \frac{1}{2}(A-B)\,, \quad S^{b\,a}_{a\,b} = \frac{1}{2}(A+B)\,,\\
S^{\alpha\beta}_{\alpha\beta} &= \frac{1}{2}(D-E)\,, \quad S^{\beta\,\alpha}_{\alpha\beta} = \frac{1}{2}(D+E)\,,\\
S^{\alpha\beta}_{a\,b} &= -\frac{1}{2}\epsilon_{ab}\epsilon^{\alpha\beta}C\,, \quad S^{a\,b}_{\alpha\beta} = -\frac{1}{2}\epsilon^{ab}\epsilon_{\alpha\beta}F\,,\\
S^{a\,\alpha}_{a\,\alpha} &= G\,, \quad S^{\alpha\,a}_{a\,\alpha} = H\,, \quad S^{a\,\alpha}_{\alpha\,a} = K\,, \quad S^{\alpha\,a}_{\alpha\,a} = L
\end{aligned}
$$

$$
\begin{aligned}
A &= S_0\frac{x_2^- - x_1^+}{x_2^+ - x_1^-}\frac{\eta_1\eta_2}{\tilde\eta_1\tilde\eta_2}\,,\\
B &= -S_0\left[\frac{x_2^- - x_1^+}{x_2^+ - x_1^-} + 2\frac{(x_1^- - x_1^+)(x_2^- - x_2^+)(x_2^- + x_1^+)}{(x_1^- - x_2^+)(x_1^-x_2^- - x_1^+x_2^+)}\right]\frac{\eta_1\eta_2}{\tilde\eta_1\tilde\eta_2}\,,\\
C &= S_0\frac{2ix_1^-x_2^-(x_1^+ - x_2^+)\eta_1\eta_2}{x_1^+x_2^+(x_1^- - x_2^+)(1 - x_1^-x_2^-)}\,, \qquad D = -S_0\,,\\
E &= S_0\left[1 - 2\frac{(x_1^- - x_1^+)(x_2^- - x_2^+)(x_1^- + x_2^+)}{(x_1^- - x_2^+)(x_1^-x_2^- - x_1^+x_2^+)}\right]\,,\\
F &= S_0\frac{2i(x_1^- - x_1^+)(x_2^- - x_2^+)(x_1^+ - x_2^+)}{(x_1^- - x_2^+)(1 - x_1^-x_2^-)\tilde\eta_1\tilde\eta_2}\,,\\
G &= S_0\frac{(x_2^- - x_1^-)}{(x_2^+ - x_1^-)}\frac{\eta_1}{\tilde\eta_1}\,, \qquad H = S_0\frac{(x_2^+ - x_2^-)}{(x_1^- - x_2^+)}\frac{\eta_1}{\tilde\eta_2}\,,\\
K &= S_0\frac{(x_1^+ - x_1^-)}{(x_1^- - x_2^+)}\frac{\eta_2}{\tilde\eta_1}\,, \qquad L = S_0\frac{(x_1^+ - x_2^+)}{(x_1^- - x_2^+)}\frac{\eta_2}{\tilde\eta_2}
\end{aligned}
$$

$$\eta_1 = \eta(p_1)e^{ip_2/2}\,, \quad \eta_2 = \eta(p_2)\,, \quad \tilde\eta_1 = \eta(p_1)\,, \quad \tilde\eta_2 = \eta(p_2)e^{ip_1/2}$$

- 주기적 경계조건



– At each crossing, S-matrix

$$e^{ip_jL}\prod_{k\neq j,1}^{N} S(p_j,p_k)=1$$

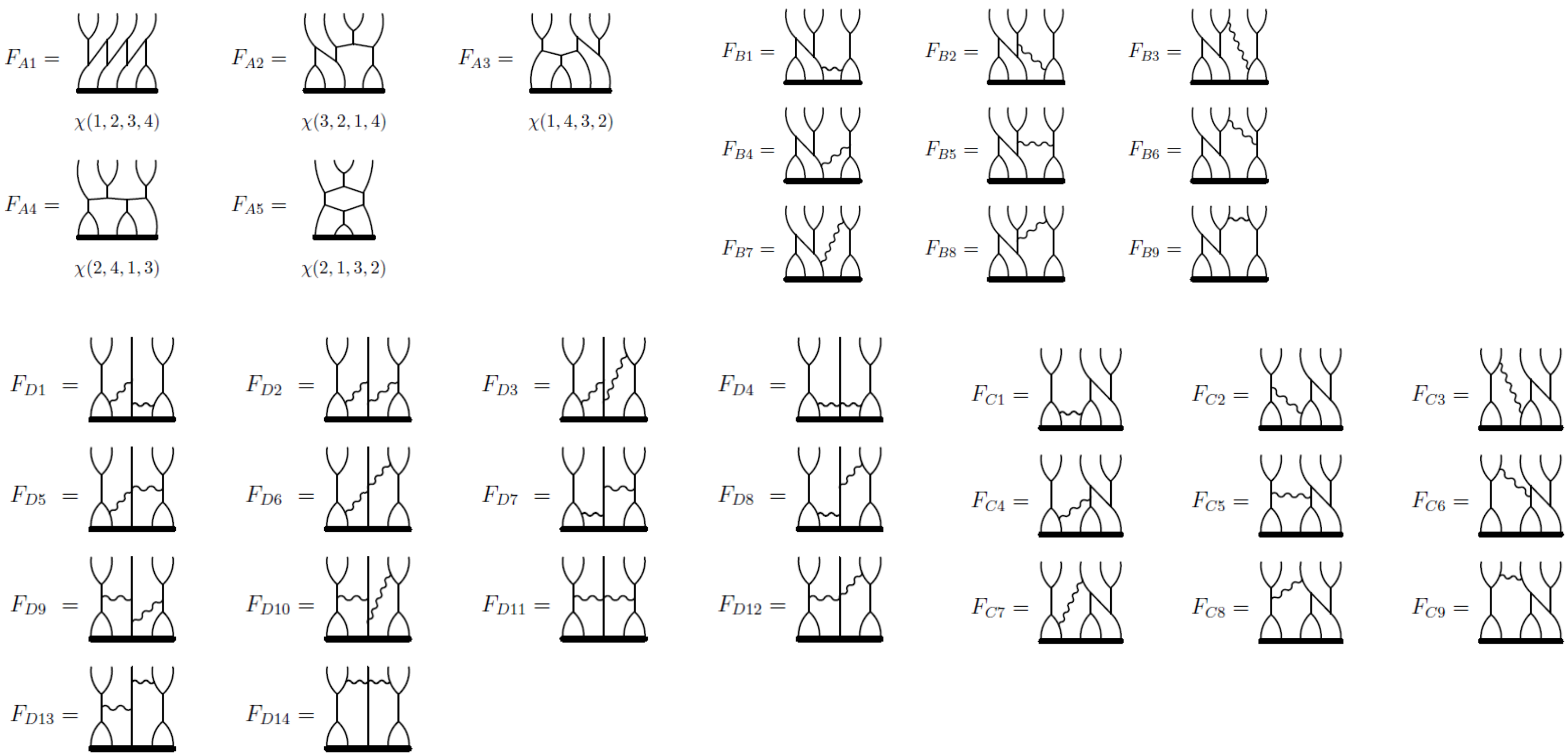
– Formulae for exact dimension can be derived

– 검증을 위해 섭동계산과 비교

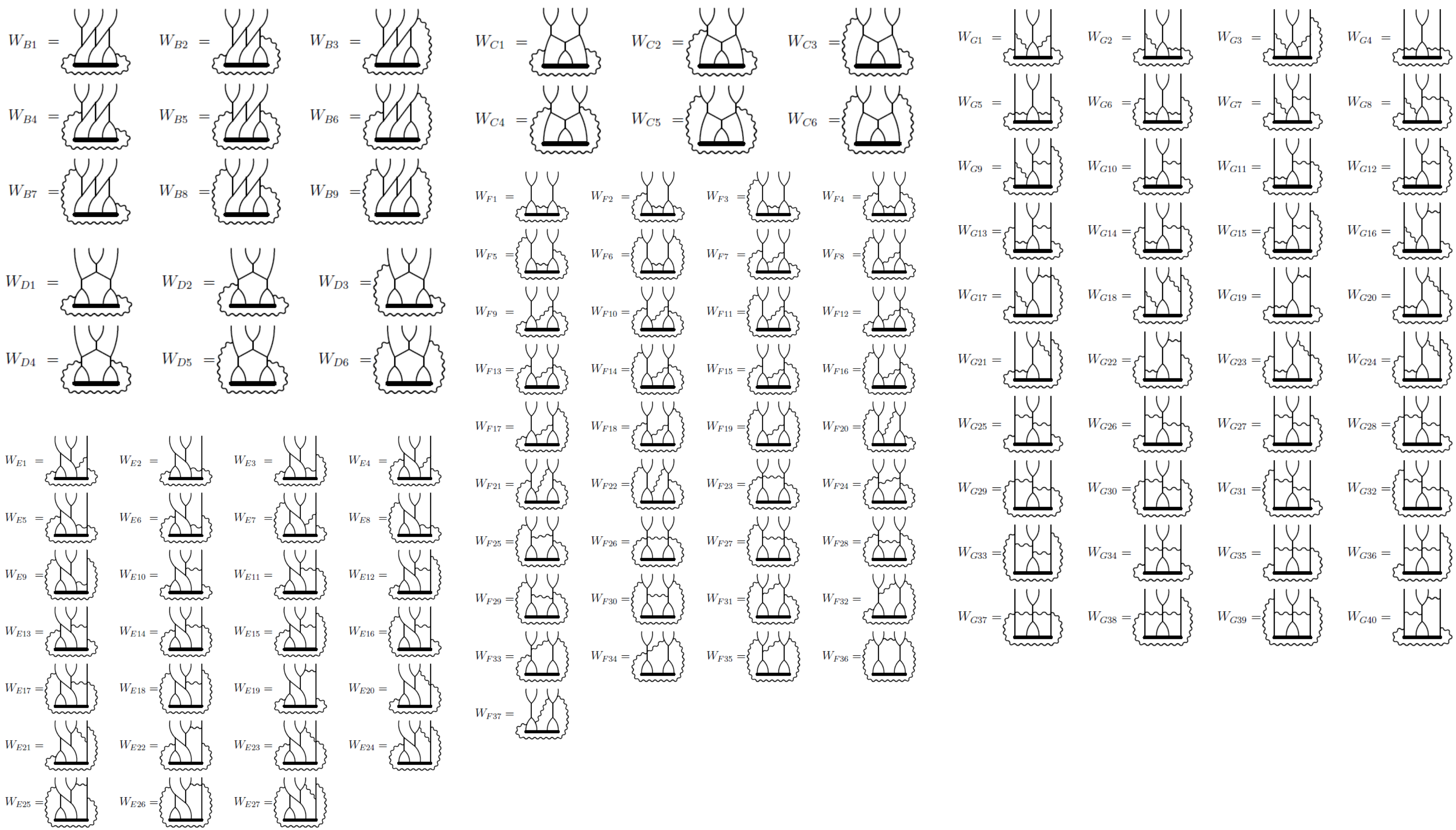
4-Loop su(2) Konishi

[Fiamberti, Santambrogio, Sieg, Zanon (2008)]

N=1 supergraphs



Wrapping diagrams



$$
\begin{aligned}
\gamma &= 4 + g^2\gamma_1 + g^4\gamma_2 + g^6\gamma_3 + g^8\gamma_4 + \ldots \\
\gamma_1 &= 6(1+\delta) \\
\gamma_2 &= -\frac{3}{\delta} - 15 - 21\delta - 9\delta^2 \\
\gamma_3 &= -\frac{3}{4\delta^3} + \frac{153}{4\delta} + 114 + \frac{495}{4}\delta + 54\delta^2 + \frac{27}{4}\delta^3 \\
\gamma_4 &= -\frac{3}{8\delta^5} + \frac{33}{2\delta^3} - \frac{1701}{4\delta} - 1230 \\
&- \frac{2427}{2}\delta - 180\delta^2 + 162\delta^4 + \frac{2997}{8}\delta^3 \\
&+ \left(-\frac{9}{\delta} + 297 + 702\delta + 234\delta^2 - 405\delta^3 - 243\delta^4\right)\zeta(3) \\
&- 360\,(1+\delta)^2\,\zeta(5)
\end{aligned}
$$

$$
\delta \equiv \frac{\sqrt{5 + 4\cos(4\pi\beta)}}{3}
$$

From nonperturbative formula

[CA-Bajnok-Bombardelli-Nepomechie (2010)]

$$\Delta = -\sum_{Q=1}^{\infty} \int_{-\infty}^{\infty} \frac{dq}{2\pi} \frac{4^L g^{2L}}{(Q^2+q^2)^L} \sum_{j,j'} (-1)^{F_{(jj')}} \left[\mathcal{S}^{(Q1)}(q,p) \mathcal{S}^{(Q1)}(q,-p) \right]_{(jj')(11)}^{(jj')(11)}$$

After residue integrals

$$\sum_{Q=1}^{\infty} \left[\frac{f_1}{Q^5} + \frac{f_2}{Q^3} + f_3(Q) \right]$$

$$f_1 = -\frac{2560(1+2u_1^2+2u_2^2)^2}{(4u_1^2+1)^2(4u_2^2+1)^2}$$

$$f_2 = \frac{\text{num}}{(4u_1^2+1)^4(4u_2^2+1)^4}$$

$$\begin{aligned}
\text{num} &= 2048\left(-1+5u_1^2+48u_1^4+96u_1^6-2u_1u_2-16u_1^3u_2-32u_1^5u_2+5u_2^2+224u_1^2u_2^2\right.\\
&+ 1024u_1^4u_2^2+1536u_1^6u_2^2+768u_1^8u_2^2-16u_1u_2^3-128u_1^3u_2^3+64u_1^8\\
&- 256u_1^5u_2^3+48u_2^4+1024u_1^2u_2^4+3200u_1^4u_2^4+2560u_1^6u_2^4-32u_1u_2^5\\
&- \left.256u_1^3u_2^5-512u_1^5u_2^5+96u_2^6+1536u_1^2u_2^6+2560u_1^4u_2^6+64u_2^8+768u_1^2u_2^8\right)
\end{aligned}$$

$$f_3 = 1775 \text{ terms}$$

$$u_{1,2} = \frac{(1-3\,\delta)^2}{2\sqrt{-1+9\,\delta^2}\left(3\sqrt{1-\delta^2}\pm 2\sqrt{\frac{3+3\delta}{1+3\,\delta}}\right)}$$

$$-(512\, i\, (-5\, i + 156\, i\, Q^{2} - 2120\, i\, Q^{4} + 16512\, i\, Q^{6} - 77952\, i\, Q^{8} + 216064\, i\, Q^{10} - 317440\, i\, Q^{12} + 196608\, i\, Q^{14} - 32768\, i\, Q^{16} - 160\, Q\, u1 + 4352\, Q^{3}\, u1 -$$
$$50432\, Q^{5}\, u1 + 321408\, Q^{7}\, u1 - 1178624\, Q^{9}\, u1 + 2396160\, Q^{11}\, u1 - 2490368\, Q^{13}\, u1 + 1277952\, Q^{15}\, u1 - 262144\, Q^{17}\, u1 - 140\, i\, u1^{2} +$$
$$6036\, i\, Q^{2}\, u1^{2} - 97280\, i\, Q^{4}\, u1^{2} + 813504\, i\, Q^{6}\, u1^{2} - 3904512\, i\, Q^{8}\, u1^{2} + 10729472\, i\, Q^{10}\, u1^{2} - 16007168\, i\, Q^{12}\, u1^{2} + 12238848\, i\, Q^{14}\, u1^{2} -$$
$$4456448\, i\, Q^{16}\, u1^{2} + 786432\, i\, Q^{18}\, u1^{2} - 3840\, Q\, u1^{3} + 109184\, Q^{3}\, u1^{3} - 1286656\, Q^{5}\, u1^{3} + 8186112\, Q^{7}\, u1^{3} - 29806080\, Q^{9}\, u1^{3} + 60510208\, Q^{11}\, u1^{3} -$$
$$65617920\, Q^{13}\, u1^{3} + 38076416\, Q^{15}\, u1^{3} - 11403264\, Q^{17}\, u1^{3} + 1048576\, Q^{19}\, u1^{3} - 1700\, i\, u1^{4} + 85248\, i\, Q^{2}\, u1^{4} - 1436928\, i\, Q^{4}\, u1^{4} +$$
$$12094720\, i\, Q^{6}\, u1^{4} - 57501696\, i\, Q^{8}\, u1^{4} + 155193344\, i\, Q^{10}\, u1^{4} - 229015552\, i\, Q^{12}\, u1^{4} + 182779904\, i\, Q^{14}\, u1^{4} - 75235328\, i\, Q^{16}\, u1^{4} +$$
$$13631488\, i\, Q^{18}\, u1^{4} - 39040\, Q\, u1^{5} + 1111040\, Q^{3}\, u1^{5} - 12920832\, Q^{5}\, u1^{5} + 80328704\, Q^{7}\, u1^{5} - 284033024\, Q^{9}\, u1^{5} + 555851776\, Q^{11}\, u1^{5} -$$
$$584318976\, Q^{13}\, u1^{5} + 322699264\, Q^{15}\, u1^{5} - 83886080\, Q^{17}\, u1^{5} + 2097152\, Q^{19}\, u1^{5} - 11680\, i\, u1^{6} + 614912\, i\, Q^{2}\, u1^{6} - 10256384\, i\, Q^{4}\, u1^{6} +$$
$$83934720\, i\, Q^{6}\, u1^{6} - 384442368\, i\, Q^{8}\, u1^{6} + 984932352\, i\, Q^{10}\, u1^{6} - 1352138752\, i\, Q^{12}\, u1^{6} + 976879616\, i\, Q^{14}\, u1^{6} - 331350016\, i\, Q^{16}\, u1^{6} +$$
$$27262976\, i\, Q^{18}\, u1^{6} - 217600\, Q\, u1^{7} + 5965824\, Q^{3}\, u1^{7} - 66535424\, Q^{5}\, u1^{7} + 393981952\, Q^{7}\, u1^{7} - 1310375936\, Q^{9}\, u1^{7} + 2346057728\, Q^{11}\, u1^{7} -$$
$$2179596288\, Q^{13}\, u1^{7} + 960495616\, Q^{15}\, u1^{7} - 148897792\, Q^{17}\, u1^{7} - 49600\, i\, u1^{8} + 2532608\, i\, Q^{2}\, u1^{8} - 40226816\, i\, Q^{4}\, u1^{8} + 310726656\, i\, Q^{6}\, u1^{8} -$$
$$1330298880\, i\, Q^{8}\, u1^{8} + 3100934144\, i\, Q^{10}\, u1^{8} - 3700162560\, i\, Q^{12}\, u1^{8} + 2134900736\, i\, Q^{14}\, u1^{8} - 486539264\, i\, Q^{16}\, u1^{8} + 8388608\, i\, Q^{18}\, u1^{8} -$$
$$716800\, Q\, u1^{9} + 18268160\, Q^{3}\, u1^{9} - 189792256\, Q^{5}\, u1^{9} + 1041408000\, Q^{7}\, u1^{9} - 3144482816\, Q^{9}\, u1^{9} + 4859232256\, Q^{11}\, u1^{9} - 3649044480\, Q^{13}\, u1^{9} +$$
$$1115684864\, Q^{15}\, u1^{9} - 67108864\, Q^{17}\, u1^{9} - 133120\, i\, u1^{10} + 6116352\, i\, Q^{2}\, u1^{10} - 89391104\, i\, Q^{4}\, u1^{10} + 633085952\, i\, Q^{6}\, u1^{10} -$$
$$2452226048\, i\, Q^{8}\, u1^{10} + 4944297984\, i\, Q^{10}\, u1^{10} - 4777312256\, i\, Q^{12}\, u1^{10} + 1935671296\, i\, Q^{14}\, u1^{10} - 234881024\, i\, Q^{16}\, u1^{10} - 1392640\, Q\, u1^{11} +$$
$$31883264\, Q^{3}\, u1^{11} - 298713088\, Q^{5}\, u1^{11} + 1471610880\, Q^{7}\, u1^{11} - 3861774336\, Q^{9}\, u1^{11} + 4783603712\, Q^{11}\, u1^{11} - 2573205504\, Q^{13}\, u1^{11} +$$
$$469762048\, Q^{15}\, u1^{11} - 220160\, i\, u1^{12} + 8364032\, i\, Q^{2}\, u1^{12} - 108068864\, i\, Q^{4}\, u1^{12} + 678658048\, i\, Q^{6}\, u1^{12} - 2279079936\, i\, Q^{8}\, u1^{12} +$$
$$3649568768\, i\, Q^{10}\, u1^{12} - 2550136832\, i\, Q^{12}\, u1^{12} + 587202560\, i\, Q^{14}\, u1^{12} - 1474560\, Q\, u1^{13} + 29360128\, Q^{3}\, u1^{13} - 237502464\, Q^{5}\, u1^{13} +$$
$$998375424\, Q^{7}\, u1^{13} - 2085617664\, Q^{9}\, u1^{13} + 1797259264\, Q^{11}\, u1^{13} - 469762048\, Q^{13}\, u1^{13} - 204800\, i\, u1^{14} + 5734400\, i\, Q^{2}\, u1^{14} -$$
$$60817408\, i\, Q^{4}\, u1^{14} + 319029248\, i\, Q^{6}\, u1^{14} - 851443712\, i\, Q^{8}\, u1^{14} + 843055104\, i\, Q^{10}\, u1^{14} - 234881024\, i\, Q^{12}\, u1^{14} - 655360\, Q\, u1^{15} +$$
$$11010048\, Q^{3}\, u1^{15} - 71303168\, Q^{5}\, u1^{15} + 222298112\, Q^{7}\, u1^{15} - 234881024\, Q^{9}\, u1^{15} + 67108864\, Q^{11}\, u1^{15} - 81920\, i\, u1^{16} + 1376256\, i\, Q^{2}\, u1^{16} -$$
$$8912896\, i\, Q^{4}\, u1^{16} + 27787264\, i\, Q^{6}\, u1^{16} - 29360128\, i\, Q^{8}\, u1^{16} + 8388608\, i\, Q^{10}\, u1^{16} - 160\, Q\, u2 + 4352\, Q^{3}\, u2 - 50432\, Q^{5}\, u2 + 321408\, Q^{7}\, u2 -$$
$$1178624\, Q^{9}\, u2 + 2396160\, Q^{11}\, u2 - 2490368\, Q^{13}\, u2 + 1277952\, Q^{15}\, u2 - 262144\, Q^{17}\, u2 + 5112\, i\, Q^{2}\, u1\, u2 - 118656\, i\, Q^{4}\, u1\, u2 + 1138048\, i\, Q^{6}\, u1\, u2 -$$
$$5663744\, i\, Q^{8}\, u1\, u2 + 15286272\, i\, Q^{10}\, u1\, u2 - 22249472\, i\, Q^{12}\, u1\, u2 + 17924096\, i\, Q^{14}\, u1\, u2 - 7602176\, i\, Q^{16}\, u1\, u2 + 1572864\, i\, Q^{18}\, u1\, u2 -$$
$$4480\, Q\, u1^{2}\, u2 + 174976\, Q^{3}\, u1^{2}\, u2 - 2408960\, Q^{5}\, u1^{2}\, u2 + 16281344\, Q^{7}\, u1^{2}\, u2 - 59152896\, Q^{9}\, u1^{2}\, u2 + 117698560\, Q^{11}\, u1^{2}\, u2 -$$
$$129744896\, Q^{13}\, u1^{2}\, u2 + 79364096\, Q^{15}\, u1^{2}\, u2 - 26869760\, Q^{17}\, u1^{2}\, u2 + 3145728\, Q^{19}\, u1^{2}\, u2 + 122688\, i\, Q^{2}\, u1^{3}\, u2 - 2995968\, i\, Q^{4}\, u1^{3}\, u2 +$$
$$29113856\, i\, Q^{6}\, u1^{3}\, u2 - 143710208\, i\, Q^{8}\, u1^{3}\, u2 + 382468096\, i\, Q^{10}\, u1^{3}\, u2 - 560791552\, i\, Q^{12}\, u1^{3}\, u2 + 468844544\, i\, Q^{14}\, u1^{3}\, u2 -$$
$$220725248\, i\, Q^{16}\, u1^{3}\, u2 + 46137344\, i\, Q^{18}\, u1^{3}\, u2 - 54400\, Q\, u1^{4}\, u2 + 2505216\, Q^{3}\, u1^{4}\, u2 - 35850240\, Q^{5}\, u1^{4}\, u2 + 241755136\, Q^{7}\, u1^{4}\, u2 -$$
$$858062848\, Q^{9}\, u1^{4}\, u2 + 1658478592\, Q^{11}\, u1^{4}\, u2 - 1806041088\, Q^{13}\, u1^{4}\, u2 + 1094189056\, Q^{15}\, u1^{4}\, u2 - 325058560\, Q^{17}\, u1^{4}\, u2 +$$
$$14680064\, Q^{19}\, u1^{4}\, u2 + 1247360\, i\, Q^{2}\, u1^{5}\, u2 - 30472192\, i\, Q^{4}\, u1^{5}\, u2 + 290317312\, i\, Q^{6}\, u1^{5}\, u2 - 1384857600\, i\, Q^{8}\, u1^{5}\, u2 +$$
$$3517972480\, i\, Q^{10}\, u1^{5}\, u2 - 4901568512\, i\, Q^{12}\, u1^{5}\, u2 + 3774087168\, i\, Q^{14}\, u1^{5}\, u2 - 1488977920\, i\, Q^{16}\, u1^{5}\, u2 + 167772160\, i\, Q^{18}\, u1^{5}\, u2 -$$
$$373760\, Q\, u1^{6}\, u2 + 18141184\, Q^{3}\, u1^{6}\, u2 - 254623744\, Q^{5}\, u1^{6}\, u2 + 1650380800\, Q^{7}\, u1^{6}\, u2 - 5530009600\, Q^{9}\, u1^{6}\, u2 + 9881518080\, Q^{11}\, u1^{6}\, u2 -$$
$$9639165952\, Q^{13}\, u1^{6}\, u2 + 4859101184\, Q^{15}\, u1^{6}\, u2 - 945815552\, Q^{17}\, u1^{6}\, u2 + 33554432\, Q^{19}\, u1^{6}\, u2 + 6952960\, i\, Q^{2}\, u1^{7}\, u2 - 162570240\, i\, Q^{4}\, u1^{7}\, u2 +$$
$$1470644224\, i\, Q^{6}\, u1^{7}\, u2 - 6562283520\, i\, Q^{8}\, u1^{7}\, u2 + 15195176960\, i\, Q^{10}\, u1^{7}\, u2 - 18733858816\, i\, Q^{12}\, u1^{7}\, u2 + 11924406272\, i\, Q^{14}\, u1^{7}\, u2 -$$
$$3439329280\, i\, Q^{16}\, u1^{7}\, u2 + 301989888\, i\, Q^{18}\, u1^{7}\, u2 - 1587200\, Q\, u1^{8}\, u2 + 74530816\, Q^{3}\, u1^{8}\, u2 - 984940544\, Q^{5}\, u1^{8}\, u2 +$$
$$5928378368\, Q^{7}\, u1^{8}\, u2 - 18003984384\, Q^{9}\, u1^{8}\, u2 + 28163702784\, Q^{11}\, u1^{8}\, u2 - 22705864704\, Q^{13}\, u1^{8}\, u2 + 8858370048\, Q^{15}\, u1^{8}\, u2 -$$
$$1325400064\, Q^{17}\, u1^{8}\, u2 + 33554432\, Q^{19}\, u1^{8}\, u2 + 22906880\, i\, Q^{2}\, u1^{9}\, u2 - 491487232\, i\, Q^{4}\, u1^{9}\, u2 + 4084072448\, i\, Q^{6}\, u1^{9}\, u2 -$$
$$16451633152\, i\, Q^{8}\, u1^{9}\, u2 + 33054523392\, i\, Q^{10}\, u1^{9}\, u2 - 33852227584\, i\, Q^{12}\, u1^{9}\, u2 + 16949182464\, i\, Q^{14}\, u1^{9}\, u2 - 3758096384\, i\, Q^{16}\, u1^{9}\, u2 +$$
$$268435456\, i\, Q^{18}\, u1^{9}\, u2 - 4259840\, Q\, u1^{10}\, u2 + 178356224\, Q^{3}\, u1^{10}\, u2 - 2139226112\, Q^{5}\, u1^{10}\, u2 + 11541086208\, Q^{7}\, u1^{10}\, u2 -$$

$30298210304\, Q^{9}\, u1^{10}\, u2 + 39352008704\, Q^{11}\, u1^{10}\, u2 - 24798822400\, Q^{13}\, u1^{10}\, u2 + 7532969984\, Q^{15}\, u1^{10}\, u2 - 939524096\, Q^{17}\, u1^{10}\, u2 +$
$44515328\, i\, Q^{2}\, u1^{11}\, u2 - 840368128\, i\, Q^{4}\, u1^{11}\, u2 + 6171262976\, i\, Q^{6}\, u1^{11}\, u2 - 21483225088\, i\, Q^{8}\, u1^{11}\, u2 + 35381051392\, i\, Q^{10}\, u1^{11}\, u2 -$
$28059893760\, i\, Q^{12}\, u1^{11}\, u2 + 10905190400\, i\, Q^{14}\, u1^{11}\, u2 - 1879048192\, i\, Q^{16}\, u1^{11}\, u2 - 7045120\, Q\, u1^{12}\, u2 + 239206400\, Q^{3}\, u1^{12}\, u2 -$
$2496135168\, Q^{5}\, u1^{12}\, u2 + 11556880384\, Q^{7}\, u1^{12}\, u2 - 24384634880\, Q^{9}\, u1^{12}\, u2 + 24289214464\, Q^{11}\, u1^{12}\, u2 - 11291066368\, Q^{13}\, u1^{12}\, u2 +$
$2348810240\, Q^{15}\, u1^{12}\, u2 + 47153152\, i\, Q^{2}\, u1^{13}\, u2 - 750256128\, i\, Q^{4}\, u1^{13}\, u2 + 4593025024\, i\, Q^{6}\, u1^{13}\, u2 - 12696158208\, i\, Q^{8}\, u1^{13}\, u2 +$
$15523119104\, i\, Q^{10}\, u1^{13}\, u2 - 8120172544\, i\, Q^{12}\, u1^{13}\, u2 + 1879048192\, i\, Q^{14}\, u1^{13}\, u2 - 6553600\, Q\, u1^{14}\, u2 + 157286400\, Q^{3}\, u1^{14}\, u2 -$
$1317011456\, Q^{5}\, u1^{14}\, u2 + 4816109568\, Q^{7}\, u1^{14}\, u2 - 6861881344\, Q^{9}\, u1^{14}\, u2 + 3841982464\, Q^{11}\, u1^{14}\, u2 - 939524096\, Q^{13}\, u1^{14}\, u2 +$
$20971520\, i\, Q^{2}\, u1^{15}\, u2 - 268435456\, i\, Q^{4}\, u1^{15}\, u2 + 1207959552\, i\, Q^{6}\, u1^{15}\, u2 - 1862270976\, i\, Q^{8}\, u1^{15}\, u2 + 1073741824\, i\, Q^{10}\, u1^{15}\, u2 -$
$268435456\, i\, Q^{12}\, u1^{15}\, u2 - 2621440\, Q\, u1^{16}\, u2 + 33554432\, Q^{3}\, u1^{16}\, u2 - 150994944\, Q^{5}\, u1^{16}\, u2 + 232783872\, Q^{7}\, u1^{16}\, u2 - 134217728\, Q^{9}\, u1^{16}\, u2 +$
$33554432\, Q^{11}\, u1^{16}\, u2 - 140\, i\, u2^{2} + 6036\, i\, Q^{2}\, u2^{2} - 97280\, i\, Q^{4}\, u2^{2} + 813504\, i\, Q^{6}\, u2^{2} - 3904512\, i\, Q^{8}\, u2^{2} + 10729472\, i\, Q^{10}\, u2^{2} -$
$16007168\, i\, Q^{12}\, u2^{2} + 12238848\, i\, Q^{14}\, u2^{2} - 4456448\, i\, Q^{16}\, u2^{2} + 786432\, i\, Q^{18}\, u2^{2} - 4480\, Q\, u1\, u2^{2} + 174976\, Q^{3}\, u1\, u2^{2} - 2408960\, Q^{5}\, u1\, u2^{2} +$
$16281344\, Q^{7}\, u1\, u2^{2} - 59152896\, Q^{9}\, u1\, u2^{2} + 117698560\, Q^{11}\, u1\, u2^{2} - 129744896\, Q^{13}\, u1\, u2^{2} + 79364096\, Q^{15}\, u1\, u2^{2} - 26869760\, Q^{17}\, u1\, u2^{2} +$
$3145728\, Q^{19}\, u1\, u2^{2} - 3880\, i\, u1^{2}\, u2^{2} + 214272\, i\, Q^{2}\, u1^{2}\, u2^{2} - 4311296\, i\, Q^{4}\, u1^{2}\, u2^{2} + 40286720\, i\, Q^{6}\, u1^{2}\, u2^{2} - 196438016\, i\, Q^{8}\, u1^{2}\, u2^{2} +$
$524476416\, i\, Q^{10}\, u1^{2}\, u2^{2} - 783220736\, i\, Q^{12}\, u1^{2}\, u2^{2} + 657063936\, i\, Q^{14}\, u1^{2}\, u2^{2} - 309329920\, i\, Q^{16}\, u1^{2}\, u2^{2} + 65011712\, i\, Q^{18}\, u1^{2}\, u2^{2} -$
$106240\, Q\, u1^{3}\, u2^{2} + 4291072\, Q^{3}\, u1^{3}\, u2^{2} - 60866560\, Q^{5}\, u1^{3}\, u2^{2} + 410987520\, Q^{7}\, u1^{3}\, u2^{2} - 1465647104\, Q^{9}\, u1^{3}\, u2^{2} + 2881683456\, Q^{11}\, u1^{3}\, u2^{2} -$
$3203268608\, Q^{13}\, u1^{3}\, u2^{2} + 2026110976\, Q^{15}\, u1^{3}\, u2^{2} - 631242752\, Q^{17}\, u1^{3}\, u2^{2} + 33554432\, Q^{19}\, u1^{3}\, u2^{2} - 46560\, i\, u1^{4}\, u2^{2} +$
$2903680\, i\, Q^{2}\, u1^{4}\, u2^{2} - 62765056\, i\, Q^{4}\, u1^{4}\, u2^{2} + 594636288\, i\, Q^{6}\, u1^{4}\, u2^{2} - 2844647424\, i\, Q^{8}\, u1^{4}\, u2^{2} + 7336697856\, i\, Q^{10}\, u1^{4}\, u2^{2} -$
$10593501184\, i\, Q^{12}\, u1^{4}\, u2^{2} + 8597929984\, i\, Q^{14}\, u1^{4}\, u2^{2} - 3594518528\, i\, Q^{16}\, u1^{4}\, u2^{2} + 450887680\, i\, Q^{18}\, u1^{4}\, u2^{2} - 1064960\, Q\, u1^{5}\, u2^{2} +$
$43145216\, Q^{3}\, u1^{5}\, u2^{2} - 6069698$ [illegible] $340\, Q^{11}\, u1^{5}\, u2^{2} -$
$25783566336\, Q^{13}\, u1^{5}\, u2^{2} + 1403$ [illegible]

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[illegible] $20\, i\, u1^{6}\, u2^{2} +$
$20476928\, i\, Q^{2}\, u1^{6}\, u2^{2} - 445663$ [illegible] $40281344\, i\, Q^{10}\, u1^{6}\, u2^{2} -$
$57707855872\, i\, Q^{12}\, u1^{6}\, u2^{2} + 40$ [illegible] $u2^{2} - 5836800\, Q\, u1^{7}\, u2^{2} +$
$231022592\, Q^{3}\, u1^{7}\, u2^{2} - 3113418752\, Q^{5}\, u1^{7}\, u2^{2} + 19156647936\, Q^{7}\, u1^{7}\, u2^{2} - 59719417856\, Q^{9}\, u1^{7}\, u2^{2} + 98927640576\, Q^{11}\, u1^{7}\, u2^{2} -$
$86576726016\, Q^{13}\, u1^{7}\, u2^{2} + 36859543552\, Q^{15}\, u1^{7}\, u2^{2} - 5687476224\, Q^{17}\, u1^{7}\, u2^{2} + 67108864\, Q^{19}\, u1^{7}\, u2^{2} - 1318400\, i\, u1^{8}\, u2^{2} +$
$83460096\, i\, Q^{2}\, u1^{8}\, u2^{2} - 1750466560\, i\, Q^{4}\, u1^{8}\, u2^{2} + 15009816576\, i\, Q^{6}\, u1^{8}\, u2^{2} - 61943316480\, i\, Q^{8}\, u1^{8}\, u2^{2} + 131372548096\, i\, Q^{10}\, u1^{8}\, u2^{2} -$
$144929980416\, i\, Q^{12}\, u1^{8}\, u2^{2} + 81040244736\, i\, Q^{14}\, u1^{8}\, u2^{2} - 19293798400\, i\, Q^{16}\, u1^{8}\, u2^{2} + 838860800\, i\, Q^{18}\, u1^{8}\, u2^{2} - 18841600\, Q\, u1^{9}\, u2^{2} +$
$712146944\, Q^{3}\, u1^{9}\, u2^{2} - 8872656896\, Q^{5}\, u1^{9}\, u2^{2} + 49515855872\, Q^{7}\, u1^{9}\, u2^{2} - 135757168640\, Q^{9}\, u1^{9}\, u2^{2} + 190264115200\, Q^{11}\, u1^{9}\, u2^{2} -$
$135780106240\, Q^{13}\, u1^{9}\, u2^{2} + 45063602176\, Q^{15}\, u1^{9}\, u2^{2} - 4362076160\, Q^{17}\, u1^{9}\, u2^{2} - 3471360\, i\, u1^{10}\, u2^{2} + 202014720\, i\, Q^{2}\, u1^{10}\, u2^{2} -$
$3918004224\, i\, Q^{4}\, u1^{10}\, u2^{2} + 30297948160\, i\, Q^{6}\, u1^{10}\, u2^{2} - 109129236480\, i\, Q^{8}\, u1^{10}\, u2^{2} + 195700981760\, i\, Q^{10}\, u1^{10}\, u2^{2} -$
$175187689472\, i\, Q^{12}\, u1^{10}\, u2^{2} + 76822872064\, i\, Q^{14}\, u1^{10}\, u2^{2} - 12683575296\, i\, Q^{16}\, u1^{10}\, u2^{2} - 35717120\, Q\, u1^{11}\, u2^{2} + 1265500160\, Q^{3}\, u1^{11}\, u2^{2} -$
$13993246720\, Q^{5}\, u1^{11}\, u2^{2} + 67825041408\, Q^{7}\, u1^{11}\, u2^{2} - 156105703424\, Q^{9}\, u1^{11}\, u2^{2} + 174785036288\, Q^{11}\, u1^{11}\, u2^{2} - 97626619904\, Q^{13}\, u1^{11}\, u2^{2} +$
$23018340352\, Q^{15}\, u1^{11}\, u2^{2} - 5611520\, i\, u1^{12}\, u2^{2} + 282558464\, i\, Q^{2}\, u1^{12}\, u2^{2} - 4798283776\, i\, Q^{4}\, u1^{12}\, u2^{2} + 32089702400\, i\, Q^{6}\, u1^{12}\, u2^{2} -$
$95158272000\, i\, Q^{8}\, u1^{12}\, u2^{2} + 134215630848\, i\, Q^{10}\, u1^{12}\, u2^{2} - 91838480384\, i\, Q^{12}\, u1^{12}\, u2^{2} + 27246198784\, i\, Q^{14}\, u1^{12}\, u2^{2} - 36700160\, Q\, u1^{13}\, u2^{2} +$
$1204289536\, Q^{3}\, u1^{13}\, u2^{2} - 11200888832\, Q^{5}\, u1^{13}\, u2^{2} + 43618664448\, Q^{7}\, u1^{13}\, u2^{2} - 77594624000\, Q^{9}\, u1^{13}\, u2^{2} + 61991813120\, Q^{11}\, u1^{13}\, u2^{2} -$
$21139292160\, Q^{13}\, u1^{13}\, u2^{2} - 5079040\, i\, u1^{14}\, u2^{2} + 206831616\, i\, Q^{2}\, u1^{14}\, u2^{2} - 2758279168\, i\, Q^{4}\, u1^{14}\, u2^{2} + 14705229824\, i\, Q^{6}\, u1^{14}\, u2^{2} -$
$32015122432\, i\, Q^{8}\, u1^{14}\, u2^{2} + 28286386176\, i\, Q^{10}\, u1^{14}\, u2^{2} - 10401873920\, i\, Q^{12}\, u1^{14}\, u2^{2} - 15728640\, Q\, u1^{15}\, u2^{2} + 476053504\, Q^{3}\, u1^{15}\, u2^{2} -$
$3422552064\, Q^{5}\, u1^{15}\, u2^{2} + 8405385216\, Q^{7}\, u1^{15}\, u2^{2} - 7784628224\, Q^{9}\, u1^{15}\, u2^{2} + 2952790016\, Q^{11}\, u1^{15}\, u2^{2} - 1966080\, i\, u1^{16}\, u2^{2} +$
$59506688\, i\, Q^{2}\, u1^{16}\, u2^{2} - 427819008\, i\, Q^{4}\, u1^{16}\, u2^{2} + 1050673152\, i\, Q^{6}\, u1^{16}\, u2^{2} - 973078528\, i\, Q^{8}\, u1^{16}\, u2^{2} + 369098752\, i\, Q^{10}\, u1^{16}\, u2^{2} -$

섭동계산결과 확인!

$$
\begin{aligned}
- & \frac{3}{8\delta^5} + \frac{33}{2\delta^3} - \frac{1701}{4\delta} - 1230 \\
- & \frac{2427}{2}\delta - 180\delta^2 + 162\delta^4 + \frac{2997}{8}\delta^3 \\
+ & \left(-\frac{9}{\delta} + 297 + 702\delta + 234\delta^2 - 405\delta^3 - 243\delta^4\right)\zeta(3) \\
- & 360\,(1+\delta)^2\,\zeta(5)
\end{aligned}
$$

Conclusion

20세기
섭동현상

비섭동현상

21세기 이론물리학은 비섭동의 시대