# Integrable Deformations of AdS/CFT

AdS/CFT is between

String theory on $AdS_d \times \underbrace{M^{10-d}}_{\text{Compact}}$

&

$(d-1)$-D (Super gauge theory) which is $\underbrace{\text{"CFT"}}$ *
such as SYM

$\lambda \leftarrow$ 't Hooft coupling

$g_s \to 0$, $N_c \to \infty$

$\boxed{\text{Planar limit}}$

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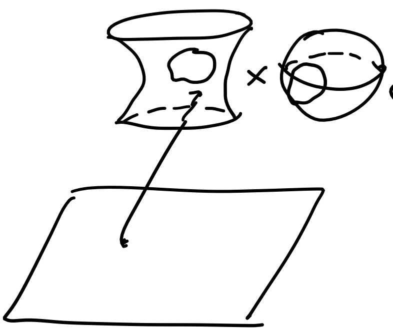
2d nLσm: $S = \frac{\sqrt{\lambda}}{4\pi} \int G_{ij}(X)\, \partial_\mu X^i \partial^\mu X^j \, d^2x$



$\leftarrow$ 10d target space

2d (non CFT) QFT

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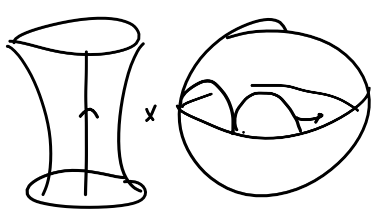
AdS/CFT relates

Energy of string states $\longleftrightarrow$ gauge-invariant operators

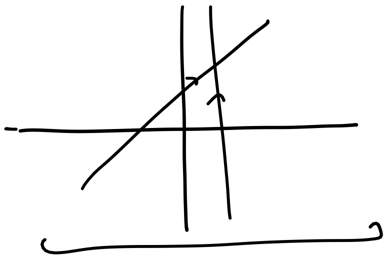
conf-dim.

spin chain

$\uparrow \downarrow \uparrow \downarrow$



$\lambda \gg 1$ $\leftarrow$



$\lambda \ll 1$ $\rightarrow$

$\mathrm{tr}(XY\psi A_\mu \cdots)$

composite op. of SYM.

string conf.
such as Giant Magnon

"Exact" tool.

(Ex)

$\underline{AdS_5 \times S^5}$

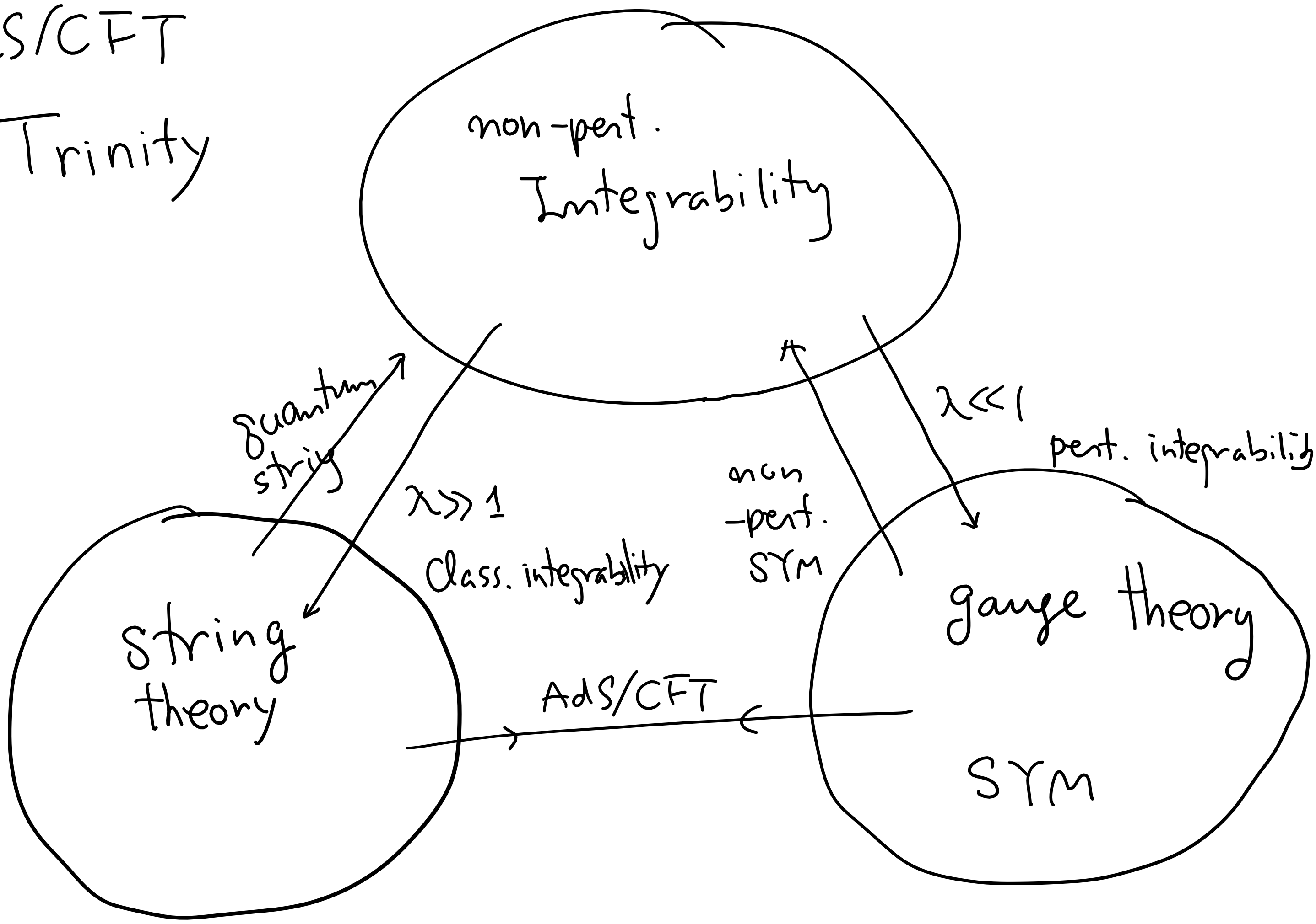
$\frac{SO(4,2)}{SO(4,1)} \times \frac{SO(6)}{SO(5)} \to \frac{PSU(2,2|4)}{SO(4,1) \times SO(5)}$ $\Longrightarrow$ $\boxed{\begin{array}{c}\text{little-group} \\ SU(2,2)^2\text{-inv.} \\ \text{Exact } S\text{-matrix}\end{array}}$ $\Leftarrow$ $\frac{N=4 \text{ SYM}}{\text{conf-Poincare}}$ $PSU(2,2|4)$ $\leftarrow$ Rsym

Exact $\downarrow$ any $\lambda$

S-matrix $\downarrow$ YBE.

AdS/CFT

Trinity

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AdS/CFT duality would be impossible to prove because String theory is solvable in "classical limit" ($\lambda \gg 1$) while gauge theory is solvable only pert. ($\lambda \ll 1$).

"2d nL$\sigma$M = integrable" is very crucial.

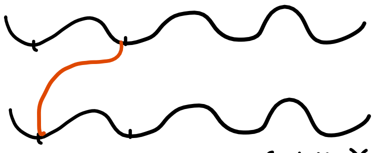
__Claim__. AdS/CFT is the duality between

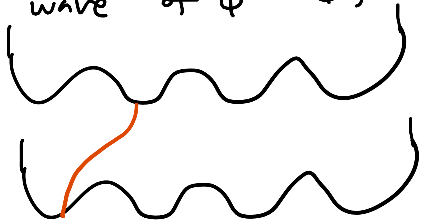
$$\boxed{\text{2D } \underline{\text{Integrable}} \text{ QFT with "target manifold" } AdS_d \times M^{10-d} \iff (d-1)\text{-D CFTs.}}$$

- naive comparisons of BPS are dangerous.
- holography is not "general".

# Integrability is Very delicate!

(ex) soliton of sine Gordon vs. solitary wave of $\phi^4 - \phi^6, \cdots$





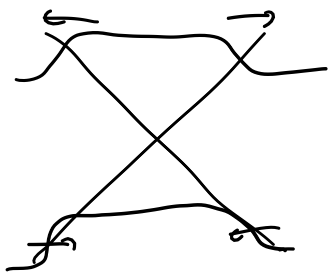
$\phi = \tan^{-1}(e^{(x-x_0)})$

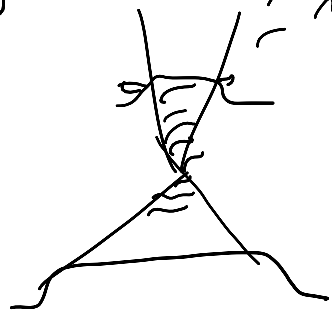
$\phi = th(x-x_0)$

both are solitary wave propagating without distortion

But interacting solitons are very different.

rad.





## How to extend "S – I – G" Trinity?

As $\lambda \gg 1 \to$ classical integrability of 2d $\sigma$ model.

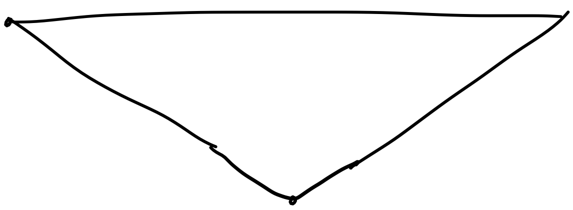
Exact S-matrix $\to$ deform S-matrix but keep integrability (YBE)

As $\lambda \ll 1 \to$ SYM or CS deformed; less susy but CFT.

## Individually deformed.

$S \to S'$

$G \to G'$



$I \to I'$

2d σM : not FP → RG flows.

<u>UV</u> ⟶ IR

flat $\mathbb{R}^d$ "compact"

pert. $\dfrac{dG_{ij}}{dt} = -\underset{\substack{\uparrow \\ \text{Ricci tensor}}}{R_{ij}} + \mathcal{O}(R^2)$ is available only in UV.

understood only nonpert. → need integrability

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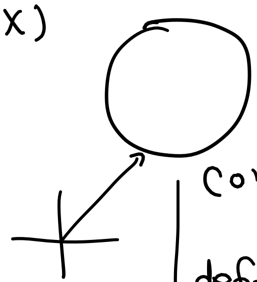
(ex) $S^1$



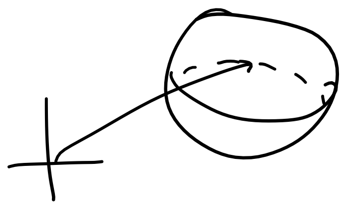
compactified boson
(free massive fermion).

<u>deform</u>

SG model (≡ massive Thirring)

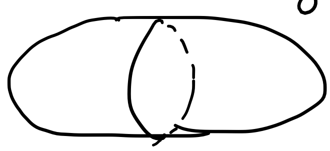
$\cos\beta\phi$ ; $\phi \to \phi + \frac{2\pi}{\beta}$

$S^2$
$(O(3))$



<u>deform</u>

Sausage



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$\hat{R}$ vs. $\hat{R}_b$

↑ classical group ↑ quantum group.

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① Drinfeld twist. → "γ-def" "β-def"

② Klimčik " . → "η-def" "non-commut."

① $\gamma$ (or $\beta$) - deformation

(1) String : Lunin-Maldacena TST.

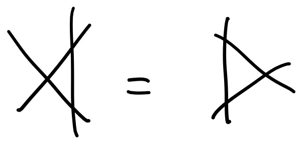
T-dual + shift in isometry ($\gamma$)
+ T-dual back.

Leigh-Strassler

(2) SYM : $V \sim |X-Y|^2 + \cdots \rightarrow |X-\beta Y|^2 + \cdots$

Spin-chain for $\lambda \ll 1$ ; $H = \sum \vec{S}_i \cdot \vec{S}_{i+1}$

(3) Int. : Drinfeld twist. (CA, Bajnok, Bomba.., Nepomechie)

$S \overset{\text{sol.}}{\Leftarrow}$  $\quad S' = F \cdot S \cdot F$ is also sol.

$F = e^{\gamma i (h \otimes 1 - 1 \otimes h)}$

proved by [CA+....]

even can be extended to 3-params $|X-\beta_1 Y|^2 + |Y-\beta_2 Z|^2 + \cdots$

$\Rightarrow$ may not be conf.

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② $\eta$- deformation

(1) string

$g = psu(2,2|4) = \underbrace{g^{(0)}}_{h = so(4,1)\times so(5)} \oplus g^{(1)} \oplus g^{(2)} \oplus g^{(3)}$

$[g^{(n)}, g^{(m)}] \subset g^{(p)}$ $\quad p = n+m \mod 4$.

$S(g/h) = \mathrm{str} \int [\gamma^{\mu\nu} j^{(2)}_\mu j^{(2)}_\nu + \epsilon^{\mu\nu} j^{(1)}_\mu j^{(3)}_\nu] d^2x$ $\qquad j^{(a)}_\mu \equiv g^{(a)-1} \partial_\mu g^{(a)}$

$\simeq P_-^{\alpha\beta} j_\alpha d(j_\beta)$ $\quad (P_\pm = \gamma^{\alpha\beta} \pm \epsilon^{\alpha\beta})$

$d = P_{(1)} + 2P_{(2)} - P_{(3)}$

$\Downarrow$

$j_\mu \, d \circ \frac{1}{1-\eta R_g \circ d} j_\nu$

(cf) $O(3) \rightarrow$ Sausage deform of target manifold.

(2) Gauge theory ; not much studied. no spin-chain.

belief : non-commutative space-time

(3) Integrability

quantum group. $\Rightarrow$ Beisert-Korotkeev S-matrix

$SU(2|2) \longrightarrow SU(2|2)_q$

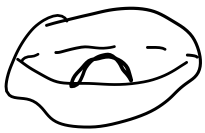
(cf) $O(3) \longrightarrow SG$

$SU(2)$ $\qquad SU(2)_q$

<u>Focus on $\lambda \gg 1$.</u>

String solutions on deformed manifold

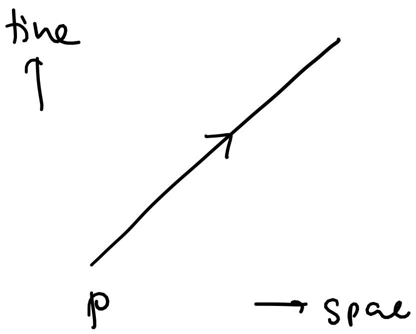
with Bozhilov.

GM  $\rightarrow \Delta E \sim \dfrac{(1+\tilde{\eta})^2}{\sqrt{1+\tilde{\eta}^2 \sin^2\frac{p}{2}}}\, e^{-\cdots}$

Can we reproduce this from $SU(2|2)_q$ S-matrix?

Yes. in 2016.11.

Finite-size effect. "Lüscher"-effect.




(no corr. if $L \to \infty$)