matlab

벡타, 행렬 연산

$\underline{a \cdot b} = a_1b_1 + \cdots$ $\qquad \vec{a} = (a_1, ---, a_N)$

$$a.*b = (a_1b_1, a_2b_2, \cdots, a_Nb_N)$$

$$a.\wedge 3 = (a_1^3, a_2^3, \cdots a_N^3)$$

$$\sin(a) = (\sin(a_1), ---, \sin(a_N))$$

$$\int_0^1 \sin(x)\,dx = \mathrm{sum}\left(\sin(x)\frac{1}{N}\right)$$

$$x = \left(0, \frac{1}{N}, ---, \frac{N}{N}\right)$$

$$L = \frac{1}{2}(\partial_\mu\phi)^2 - \frac{1}{2}(\phi^2-1)^2$$

$$\partial_\mu^2\phi = -2\phi(\phi^2-1)$$

$$\partial_t^2\phi - \partial_x^2\phi + 2(\phi^3-\phi) = 0$$

$$\phi = \phi(x-vt)$$

$$\underbrace{\phi'(v^2-1)\phi''}{} + 2(\phi^3-\phi)\phi' = 0$$

$$(v^2-1)\frac{1}{2}((\phi')^2)' + \frac{1}{2}(\phi^4-\phi^2)' = 0$$

$x \to \pm\infty$, $\phi' = 0$, $\phi = \pm 1$

$$(v^2-1)\phi'^2 + \phi^4 - \phi^2 = \text{const.} = 0$$

$$\downarrow$$

$$\left(\frac{d\phi}{dx}\right) = \frac{\phi\sqrt{(1-\phi^2)}}{\sqrt{v^2-1}} \to \int\frac{d\phi}{\phi\sqrt{1-\phi^2}} = \int\frac{dx}{\sqrt{v^2-1}}$$

$$1-\phi^2 = 1-\tanh^2 y = \frac{1}{\cosh^2 y}$$

$$\phi = \tanh y$$

$$d\phi = \frac{dy}{\cosh^2 y}$$

$$\int \frac{dy/\cosh^2 y}{\tanh y \; \frac{1}{\cosh y}} = \frac{x-x_0}{\sqrt{v^2-1}}$$

$$\frac{d\phi}{dx} = \sqrt{2U} \qquad U = \frac{1}{4}(\phi^2-1)^2$$

$$= \frac{1}{\sqrt{2}}(\phi^2 - 1)$$

$$\frac{1}{\phi-1} - \frac{1}{\phi+1} \qquad \int \frac{d\phi}{\phi^2-1} = \int dx = \frac{x-x_0}{\sqrt{2}}$$

$$= \frac{1}{2}\ln\frac{\phi-1}{\phi+1} \rightarrow \frac{\phi-1}{\phi+1} = e^{2\frac{(x-x_0)}{\sqrt{2}}}$$

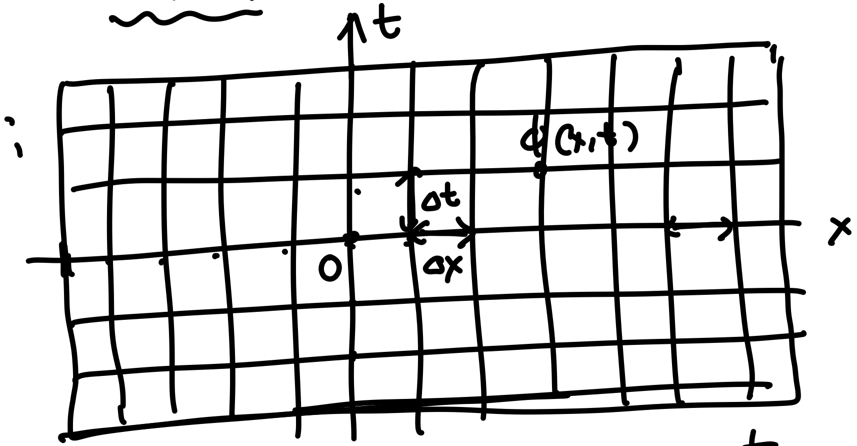
$$\phi = \frac{e + 1}{1 - e} = \pm\tanh\frac{(x-x_0)}{\sqrt{2}}.$$

$$\rightarrow \boxed{\partial_t^2\phi - \partial_x^2\phi + \phi(\phi^2-1) = 0}$$

$$\phi(x,t) = \tanh\left(\frac{x - vt - x_0}{\sqrt{2}\sqrt{1-v^2}}\right)$$

$$\partial_t^2 \phi - \partial_x^2 \phi + \phi(\phi^2 - 1) = 0$$

discretize :



$\Delta x, \Delta t \to 0$

$$\phi(x,t) \longrightarrow \phi_i^n$$

$n \leftarrow t$

$i \leftarrow x$

ϕ'

$$\partial_x \phi(x) = \frac{\phi(x+\Delta x) - \phi(x)}{\Delta x} = \frac{\phi_{i+1}^n - \phi_i^n}{\Delta x}$$

$i \to i+1$

$$\frac{\phi_{i+2}^n + \phi_i^n - 2\phi_{i+1}^n}{\Delta x^2}$$

$$\partial_t \phi = \frac{\phi_i^{n+1} - \phi_i^n}{\Delta t}$$

$$\partial_x^2 \phi = \frac{\phi'(x+\Delta x) - \phi'(x)}{\Delta x} = \frac{\left(\frac{\phi_{i+2}^n - \phi_{i+1}^n}{\Delta x}\right) - \left(\frac{\phi_{i+1}^n - \phi_i^n}{\Delta x}\right)}{\Delta x}$$

$$\partial_t^2 \phi \rightarrow \frac{\phi_i^{n+2} + \phi_i^{n} - 2\phi_i^{n+1}}{(\Delta t)^2}$$

$$\partial_t^2 \phi - \partial_x^2 \phi + \phi^3 - \phi = 0$$

$$\boxed{\frac{\phi_i^{n+1} + \phi_i^{n-1} - 2\phi_i^{n}}{\Delta t^2} - \frac{\phi_{i+1}^{n} + \phi_{i-1}^{n} - 2\phi_i^{n}}{\Delta x^2} + {\phi_i^{n}}^3 - \phi_i^{n} = 0}$$

$$\phi_i^{n \in 1\ 2\ 3\ \cdots}$$

초기조건

$$t=0 \rightarrow \phi(x, t=0) = \phi_i^{1}$$

$$\dot{\phi}(x, t=0) = \frac{\phi_i^{2} - \phi_i^{1}}{\Delta t} \Rightarrow \phi_i^{2}$$

$$\phi_i^{n+1} = 2\phi_i^{n} - \phi_i^{n-1} + \frac{\Delta t^2}{\Delta x^2}\left(\phi_{i+1}^{n} + \phi_{i-1}^{n} - 2\phi_i^{n}\right) + \left(\phi_i^{n} - {\phi_i^{n}}^3\right)\Delta t^2$$

$$(\vec{v})_{i+1} = (\quad)_i$$

(ex)
$$\begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} \longrightarrow \begin{pmatrix} v_2 \\ v_3 \\ v_1 \end{pmatrix}$$

$$L = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}, \quad \underbrace{\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}}_{L} \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} = \begin{pmatrix} v_2 \\ v_3 \\ v_1 \end{pmatrix}$$

$$(L \cdot \vec{v})_i = \vec{v}_{i+1}$$

$$R = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} = \begin{pmatrix} v_3 \\ v_1 \\ v_2 \end{pmatrix} \qquad (R \cdot \vec{v})_i = \vec{v}_{i-1}$$

$$\phi_i^{n+1} = 2\phi_i^{n} - \phi_i^{n-1} + \frac{\Delta t^2}{\Delta x^2}\left(\phi_{i+1}^{n} + \phi_{i-1}^{n} - 2\phi_i^{n}\right) + \left(\phi_i^{n} - {\phi_i^{n}}^{3}\right)\Delta t^2$$

$$\vec{\phi}^{(n+2)} = 2\vec{\phi}^{(n+1)} - \vec{\phi}^{(n)} + \frac{\Delta t^2}{\Delta x^2}\left(L\cdot\vec{\phi}^{(n+1)} + R\cdot\vec{\phi}^{(n+1)} - 2\vec{\phi}^{(n)}\right) + \left(\vec{\phi}^{(n+1)} - {\vec{\phi}^{(n+1)}}^{3}\right)\Delta t^2$$