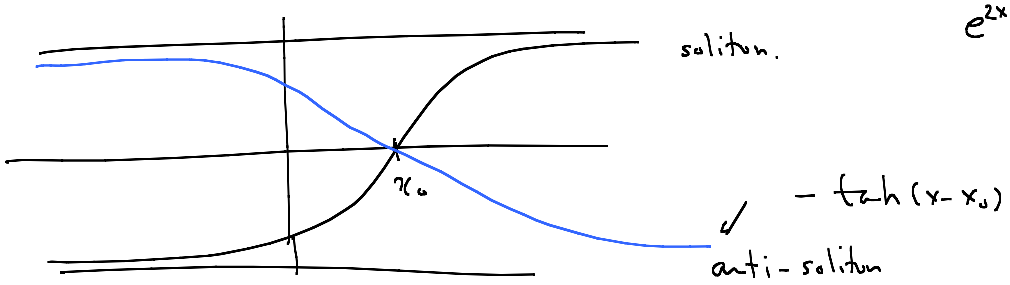
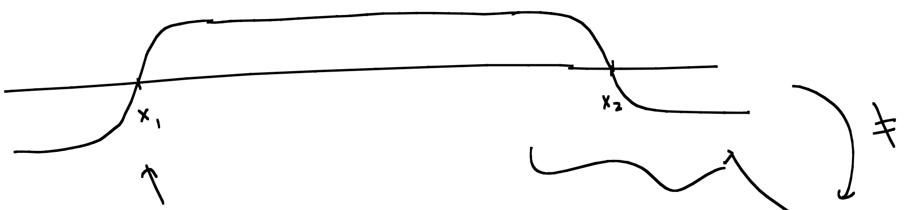
$$\frac{d\varphi}{dx} = \varphi' = \sqrt{2V(\varphi)} \qquad V = \frac{1}{2}(\varphi^2-1)^2$$

$$= 1-\varphi^2 \qquad -1 \le \varphi \le 1$$

$$\int \frac{d\varphi}{1-\varphi^2} = \int dx \longrightarrow \varphi = \tanh(x-x_0)$$

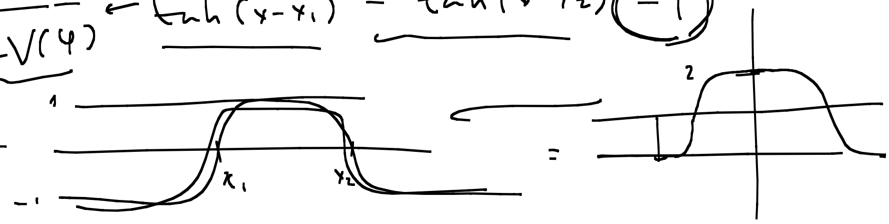
$$= \frac{e^{2x}-1}{e^{2x}+1}$$





$$\varphi' = \sqrt{2V(\varphi)} \leftarrow \tanh(x-x_1) - \tanh(x-x_2) - 1$$

nonlinear



2개 이상의 solition의 해는 찾기가 매우 어렵다.

근사적 ($x_2 >> x_1$) 인 경우 sol. ⟷ anti-sol. 사이의 힘?

$$\boxed{\mathcal{L} = \frac{1}{2}(\partial_\mu\varphi)^2 - V(\varphi)} \longrightarrow \varphi(x,t)$$

Euler Lagrange Eq: $\partial_\mu\partial^\mu\varphi + V'(\varphi) = 0$

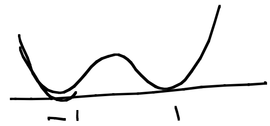
$$\ddot{\varphi} = 0 \leftarrow \ddot{\varphi} - \varphi'' + V'(\varphi) = 0$$

$$\varphi'' = V'(\varphi) \qquad \varphi(t,x) = \varphi(x)$$

$$\left(\frac{1}{2}\varphi'^2\right)' = \varphi'\varphi'' = \varphi' V'(\varphi) = V(\varphi)' \quad \frac{d}{dx}$$

$$\leftarrow \Big(\underbrace{\frac{1}{2}\varphi'^2 - V}_{=C=0}\Big)' = 0$$

$\varphi'^2 = 2V \to \varphi' = \sqrt{2V}$ $\quad V = (\varphi^2-1)^2$



$x \to \pm\infty$ $\quad \varphi \to 1$

$\varphi' \to 0$

Symmetry $\to$ Conserved quantity

$x \to x+a \quad \longrightarrow \quad$ momentum 보존

$$\underbrace{L = \frac{1}{2} m \dot{x}^2} - \cancel{V(x)}$$

$x \to x+a \quad ; \quad V(x) \to V(x+a) \neq V(x)$

$\dot{x} \to \dot{x} + \cancel{\dot{a}}^{=0}$

$$L = \frac{1}{2}(\partial_\mu \varphi)^2 - V(\varphi)$$

$x^{\mu} \to x^\mu + a^\mu$ ($\mu = 0, 1$) $\quad$ L 이 변하지 않는다 : 대칭성

$$\varphi(x^\mu) \to \varphi(x^\mu + a^\mu) \underset{a \ll 1}{=} \varphi(x^\mu) + \underbrace{a^\mu \partial_\mu \varphi}_{\delta\varphi}$$

$\varphi \to \varphi + \delta\varphi \Rightarrow \partial_\mu\varphi \to \partial_\mu\varphi + \partial_\mu\delta\varphi$

$$L = L(\varphi, \partial_\mu \varphi) \Rightarrow L(\varphi + \delta\varphi, \partial_\mu\varphi + \partial_\mu\delta\varphi)$$

$$= L(\varphi, \partial_\mu\varphi) + \boxed{\frac{\partial L}{\partial \varphi}\delta\varphi + \frac{\partial L}{\partial(\partial_\mu\varphi)}\partial_\mu\delta\varphi}$$

E-L Eq. $\quad \dfrac{\partial L}{\partial \varphi} = \partial_\mu \left(\dfrac{\partial L}{\partial(\partial_\mu\varphi)}\right)$

$\ddot{\varphi} - \varphi'' + V' = 0$

$$\partial_\mu \frac{\partial L}{\partial(\partial_\mu\varphi)}\delta\varphi + \frac{\partial L}{\partial(\partial_\mu\varphi)}\partial_\mu\delta\varphi$$

$$= \partial_\mu\left(\frac{\partial L}{\partial(\partial_\mu\varphi)}\delta\varphi\right)$$

$\delta\varphi \to \underline{a\cdot\partial\varphi}$

$\delta\varphi \equiv a\cdot\partial\varphi$

$\varphi \to \varphi + a\cdot\partial\varphi$

$\mathcal{L} \to \mathcal{L} + a\cdot\partial\mathcal{L}$

$\| \quad a^\mu \cdot \partial_\mu \mathcal{L} \qquad a^\nu\partial_\nu\varphi$

$a^0 \dfrac{\partial}{\partial t} + a^1 \dfrac{\partial}{\partial x}$

$$0 = \partial_\mu\left(\frac{\partial L}{\partial(\partial_\mu\varphi)}\underbrace{\delta\varphi}_{a^\nu\partial_\nu\varphi} - \underbrace{a^\mu}_{a^\nu\delta^\mu_\nu}\mathcal{L}\right)$$

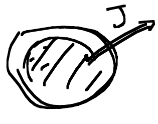
Kronecker delta

$$\delta^\mu_\nu = \begin{cases} 1 & \nu = \mu \\ 0 & \nu \neq \mu \end{cases}$$

$$= a_\nu \underbrace{\partial_\mu\left(\frac{\partial L}{\partial(\partial_\mu\varphi)}\partial\nu\varphi - \delta^\mu_\nu\mathcal{L}\right)}_{=0}$$

$a^2 = \underbrace{a^1\delta^2_1}_{=0} + a^2\underbrace{\delta^2_2}_{1}$

$\longrightarrow$ 보존법칙 $\quad \partial_\mu J^\mu = 0$



$$\dot{Q} + \underline{\vec{\nabla}\cdot\vec{J}} = 0$$

$\partial_\mu = \left(\dfrac{\partial}{\partial t}, \vec{\nabla}\right)$

$$T^{\mu}{}_{\nu} = \frac{\partial L}{\partial(\partial_\mu \varphi)} \partial_\nu \varphi - \delta^{\mu}_{\nu} \mathcal{L} \quad : \text{Stress Energy Tensor}$$

$$T^0{}_0 = \frac{\partial L}{\partial(\partial_0 \varphi)} \dot\varphi - \mathcal{L} = \dot\varphi^2 - \left(\frac{1}{2}(\dot\varphi^2 - \varphi'^2) - V\right)$$

$$= \frac{1}{2}\left(\dot\varphi^2 + \varphi'^2\right) + V$$

$$= \mathcal{H} : \text{energy density}$$

$\partial(\partial_0\varphi) = \dot\varphi$

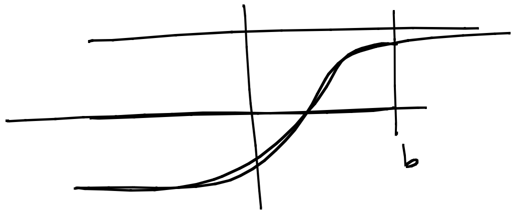
$$\mathcal{L} = \frac{1}{2}(\partial_\mu\varphi)^2 - V = \frac{1}{2}(\dot\varphi^2 - \varphi'^2) - V$$

$$\partial_1 \varphi = \frac{d\varphi}{dx} = \varphi' \qquad \partial_0\varphi = \frac{\partial \varphi}{\partial t} = \dot\varphi$$

momentum

$$T^0{}_1 = \dot\varphi\varphi'(x) = -p$$

$$p = -\int_{-\infty}^{b} \dot\varphi\,\varphi'\,dx$$



$$F = \frac{dp}{dt} = -\int_{-\infty}^{b} \underbrace{\ddot\varphi}_{\varphi'' - V'(\varphi)}\varphi' + \underbrace{\dot\varphi\,\dot\varphi'}_{(\frac{1}{2}\dot\varphi^2)'}\,dx$$

$$\ddot\varphi - \varphi'' + V'(\varphi) = 0$$

$$\varphi'\varphi'' = \left(\frac{1}{2}\varphi'^2\right)' \qquad V'(\varphi)\varphi' = \frac{dV}{dx}$$

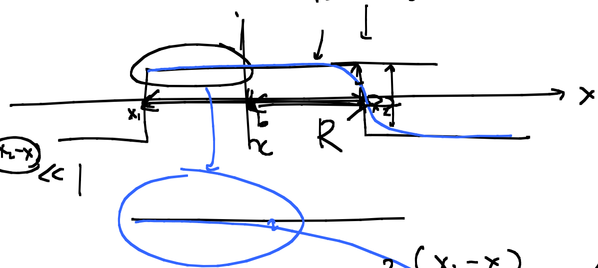
$$= -\left(\frac{1}{2}\varphi'^2\right)\Big|_{-\infty}^{b} + V\Big|_{-\infty}^{b} - \frac{1}{2}\dot\varphi^2\Big|_{-\infty}^{b}$$

$\varphi \to -1$, $\dot\varphi = \varphi' \to 0$, $V \to 0$

$$\underline{F = -\frac{1}{2}(\varphi'^2 + \dot\varphi^2) + V\Big|_{x=b}}$$

$$\varphi = \varphi_1 + \underbrace{\varphi_2 - 1}_{\Delta \ll 1}$$

$\varphi_2 = th(x_2 - x)$



$e^{-2(x_2-x)} \ll 1$

$$\varphi_2 = \frac{1 - e^{-2(x_2-x)}}{1 + \underbrace{e^{-2(x_2-x)}}_{\epsilon}} \approx 1 - 2\epsilon = 1 - 2e^{-2(x_2-x)} \approx 1$$

$$\boxed{\varphi_2 - 1 = -2e^{-2(x_2-x)} \equiv \Delta}$$

$$\Delta' = -4e^{-2(x_2-x)}$$

$(\varphi_1' + \Delta')^2$

$$F = -\frac{1}{2}(\varphi'^2 + \dot\varphi^2) + V \Big| \qquad \varphi = \varphi_1 + \Delta$$

$$= -\frac{1}{2}(\varphi_1'^2 + \dot\varphi_1^2) + V(\varphi_1) \qquad V(\varphi_1 + \Delta)$$

$$-\varphi_1'\Delta' + V'(\varphi_1)\Delta$$

$$\varphi_1 = th(x - x_1) \quad \ll 1$$

$$= \frac{1 - e^{-2(x-x_1)}}{1 + e^{-2(x-x_1)}} = 1 - 2\underbrace{e^{-2(x-x_1)}}_{\alpha}$$

$$\varphi_1' = 4e^{-2(x-x_1)}$$

$$-\varphi_1'\Delta' = 16\,e^{-2(x-x_1)}e^{-2(x_2-x)}$$

$V = (\varphi^2-1)^2$

$V' = 4\varphi(\varphi^2-1) = -16\, e^{-2(x-x_1)}\, e^{-2(x_2-x_1)} = e^{-2R}$

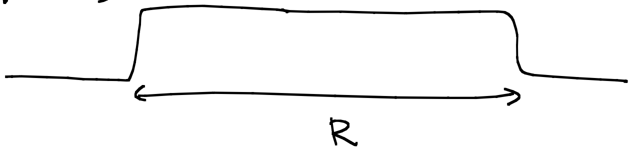
$(1-2) - 4\alpha$

≈ 1

$\Delta = -2\, e^{-2(x_2-x)}$

$\therefore V'\Delta = 32\, e^{-2R}$ $\Rightarrow F(R) = 48\, e^{-2R}$

$V = (\varphi^{\cancel{2}}-1)^2$



H.W.

I. ① $V = \varphi^2(\varphi^2-1)^2$ 일때 $F_{SA} = ?$

φ^6

↓ S, A

② $V = (\varphi^2-1)^2(\varphi^2-4)^2$ $F_{SA} = ?$

φ^8

II. 인터넷에서 솔리톤의 응용을 찾아볼것.

① 광섬유, 광통신 : 솔리톤 함수

② 신경 신호전달

③ Protein (단백질)

④ 2 회.