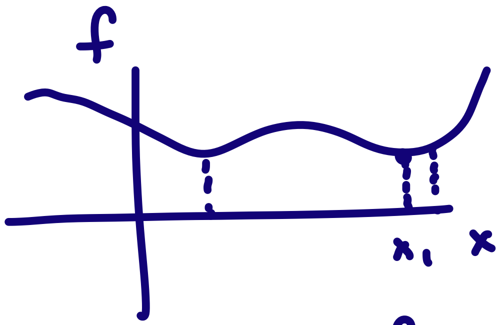
$$L = T - V = L(q_i(t), \dot{q}_i(t))$$

↑ scalar

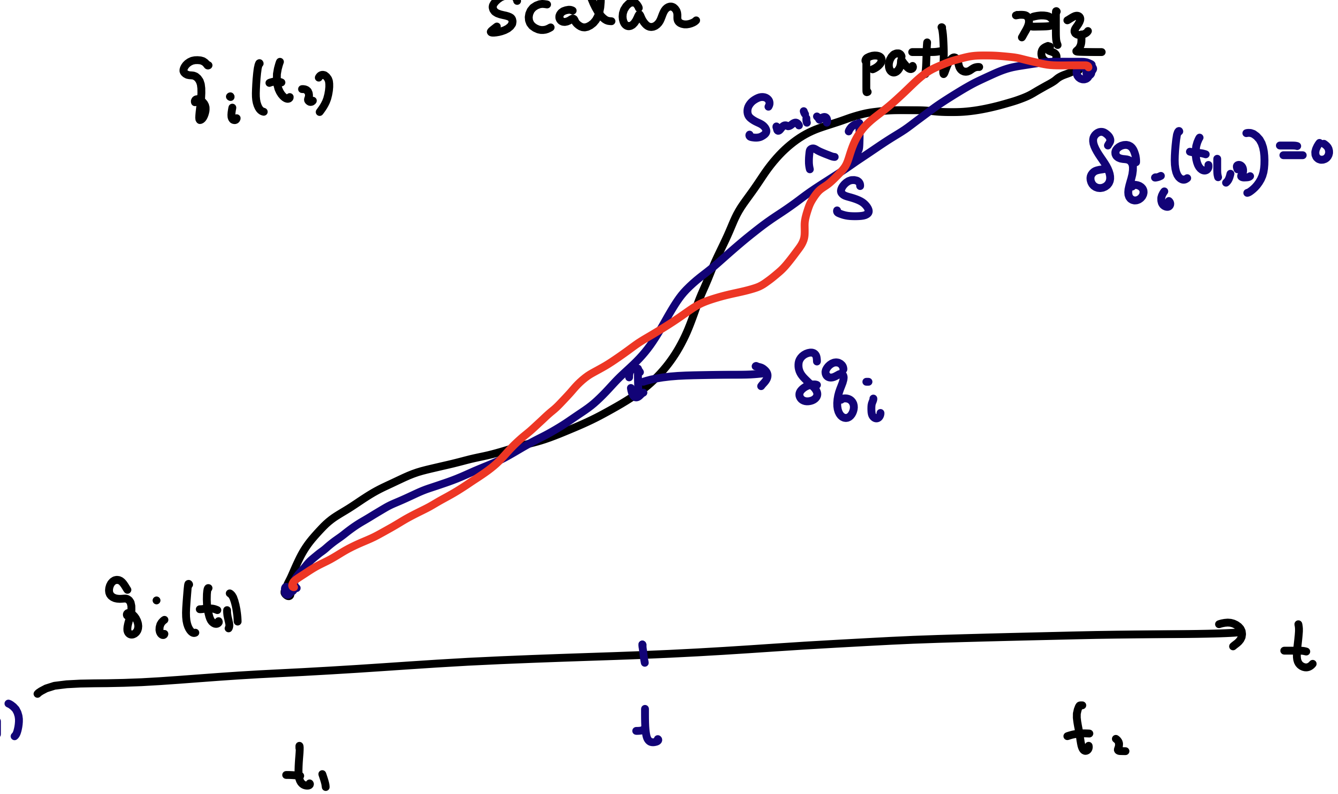
$$S = \int_{t_1}^{t_2} L \, dt$$



$$f(x_1 + \eta) \geq f(x_1)$$

$$\underset{=0}{\boxed{f'(x_1)}}\,\eta = 0$$

$$\boxed{\delta S\Big|_{q_i} = 0}$$



$$\delta S = S(q + \delta q) - S(q)$$

$$= \int_{t_1}^{t_2} L(q_i + \delta q_i, \dot{q}_i + \delta \dot{q}_i)\, dt - \int_{t_1}^{t_2} L(q_i, \dot{q}_i)\, dt$$

$$\delta S = \int_{t_1}^{t_2} \underbrace{L(q_i+\delta q_i, \dot{q}_i+\delta\dot{q}_i)}_{\downarrow} dt - \int_{t_1}^{t_2} \cancel{L(q_i,\dot{q}_i)}\, dt$$

$$\cancel{L} + \sum_{i=1}^{N}\left(\frac{\partial L}{\partial q_i}\delta q_i + \frac{\partial L}{\partial \dot{q}_i}\delta\dot{q}_i\right)$$

$$= \sum_{i=1}^{N} \int \left(\frac{\partial L}{\partial q_i}\delta q_i + \frac{\partial L}{\partial \dot{q}_i}\underbrace{\delta\dot{q}_i}_{\frac{d}{dt}\delta q_i}\right) dt$$

$$= \sum_{i=1}^{N}\left[\underbrace{\int_{t_1}^{t_2}\left(\frac{\partial L}{\partial q_i} - \frac{d}{dt}\frac{\partial L}{\partial \dot{q}_i}\right)\delta q_i \, dt}_{=0} + \underbrace{\frac{\partial L}{\partial \dot{q}_i}\delta q_i\Big|_{t_1}^{t_2}}_{=0 \ \because\ \delta q_i(t_{1,2})=0}\right]$$

for any δq_i $\rightarrow$ $\underline{\dfrac{\partial L}{\partial q_i} - \dfrac{d}{dt}\dfrac{\partial L}{\partial \dot{q}_i} = 0}$

E-L 방정식

Hamilton $\longleftrightarrow$ $L(q, \dot{q})$

$$H(p, q) = p_i \dot{q}_i - L$$

$$p_i \equiv \frac{\partial L}{\partial \dot{q}_i}$$ "generalized momentum"

$$\updownarrow$$

$$\{p_i\} \longleftrightarrow \{\dot{q}_i\}$$

Hamilton Eq.

$$\frac{\partial H}{\partial p_i} = \dot{q}_i, \quad \frac{\partial H}{\partial q_i} = -\dot{p}_i \quad \Longleftrightarrow \quad \frac{\partial L}{\partial q_i} - \frac{d}{dt}\frac{\partial L}{\partial \dot{q}_i} = 0$$

$$\frac{\partial H}{\partial q_i} = -\frac{\partial L}{\partial q_i} = -\frac{d}{dt}\overbrace{\frac{\partial L}{\partial \dot{q}_i}}^{p_i} = -\dot{p}_i$$

Q.M.

H 방법이 주로 사용 : $\hat{H}\psi = i\hbar \frac{\partial \psi}{\partial t}$

$$\hat{H}(\hat{p}, \hat{q}) = \frac{\hat{p}^2}{2m} + V(\hat{q})$$

Schrödinger

Heisenberg $\longleftrightarrow$ $\frac{d}{dt}\hat{Q} = \frac{1}{i\hbar}[\hat{H}, \hat{Q}]$

Poisson Bracket

$$\frac{\partial H}{\partial q_i} = -\dot{p}_i \qquad \frac{\partial H}{\partial p_i} = \dot{q}_i$$

$$\{p_i, q_j\}_{PB} \equiv \delta_{ij} \quad , \quad \{A(p,q), B(p,q)\} \equiv \sum_{i=1}^{N}\left(\frac{\partial A}{\partial p_i}\frac{\partial B}{\partial q_i} - \frac{\partial A}{\partial q_i}\frac{\partial B}{\partial p_i}\right)$$

$$\boxed{\dot{Q} = \{H, Q\}}$$

$$Q = Q(q_i, p_i)$$

$$= \sum_i \left(\underset{\dot{q}_i}{\frac{\partial H}{\partial p_i}}\frac{\partial Q}{\partial q_i} - \underbrace{\frac{\partial H}{\partial q_i}}_{-\dot{p}_i}\frac{\partial Q}{\partial p_i}\right)$$

(cf)

$$\frac{d}{dt}\hat{Q} = \frac{1}{i\hbar}[\hat{H}, \hat{Q}]$$

$$= \underline{\sum_i \left(\frac{\partial Q}{\partial q_i}\frac{dq_i}{dt} + \frac{\partial Q}{\partial p_i}\frac{dp_i}{dt}\right)} = \frac{dQ}{dt} = \dot{Q}$$

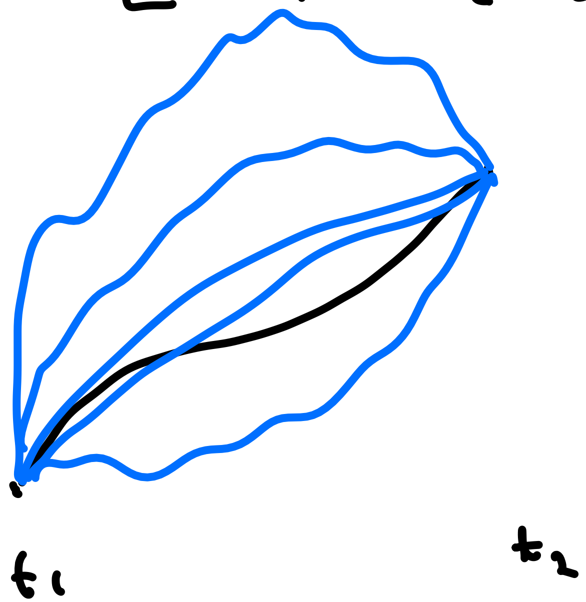
$$\lim_{\hbar \to 0} \frac{1}{i\hbar}[\ ,\ ] = \{\ ,\ \}$$

$$\{p, q\} = 1$$

H.

$$\frac{1}{i\hbar}[p, q] = 1$$

Q.M. L ? [Dirac, Feynman : Path integral]



$\delta S = 0 \Leftrightarrow$ 최소 작용량의 원리.

모든 path가 가능.

단, $\hbar \to 0 \longrightarrow \frac{\hbar}{S} \to 0$ 만 중요.

S 와 단위가 같다.

$\underline{S >>>> \hbar}$

$$M\left(\frac{L}{T}\right)^2 \cdot T = ML^2T^{-1} = MLT^{-1} \cdot L$$

Path Integral

$$\frac{\hbar}{S} \to 0 \Leftrightarrow \psi = \underbrace{\int e^{i\frac{S[\text{path}]}{\hbar}} \, d[\text{path}]}_{N=\frac{1}{\hbar}}$$

Steepest descendant method.

매우 큰수

$$\int_{-\infty}^{\infty} e^{iNf(x)} \, dx \approx e^{iNf(x_0)} + O\left(\frac{1}{N}\right)$$

$$f(x) = f(x_0) + \cancel{f'(x_0)(x-x_0)} + \frac{f''}{2}(x_0)(x-x_0)^2 + \cdots$$

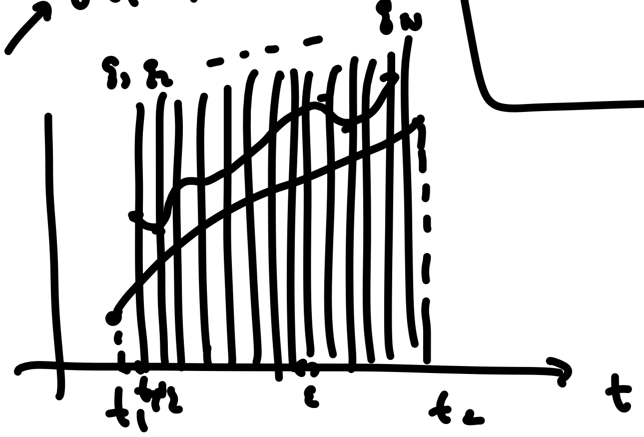
$$e^{iNf(x_0)} \cdot \underbrace{e^{iN\frac{f''}{2}(x-x_0)^2 + \cdots}}_{1}$$

quanm field theory

$\uparrow$

$q \to q_i \leftarrow x$

$q_i \to q(x,t)$



$$\int d[\text{path}] = \lim_{\epsilon \to 0} \int \prod_{i=1}^{N} dq_i$$

$$N = \frac{t_2 - t_1}{\varepsilon}$$

보존전하.

$$\frac{dQ}{dt} = 0 \rightarrow Q(t) = Q(0)$$
$$= \{H, Q\}$$

가적계 (Integrable system) $\Longleftrightarrow$ soliton

자유도 (N) = 보존전하의 갯수

$q_1 \cdots q_N$

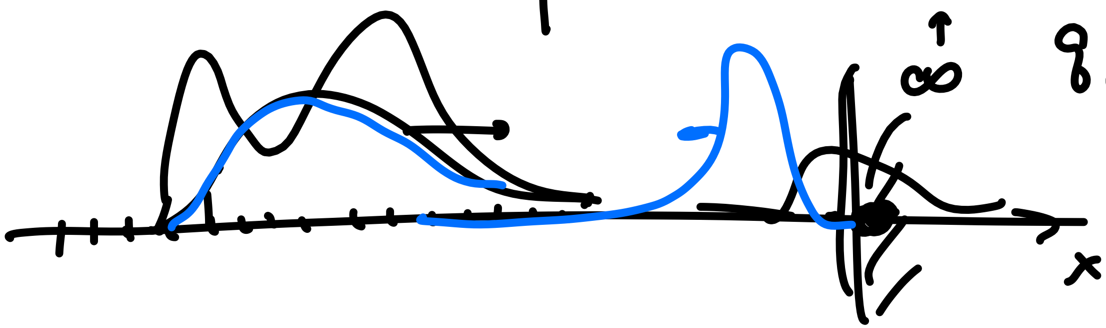
$$Q = \{ \underset{\underset{H}{\cup}}{I_i} \} \rightarrow \{H, I_i\} = 0 , \{I_i, I_j\} = 0$$

$$H = H(I, \varphi)$$

$$\underset{\text{상수}}{\omega_i(I)} = \{H, \varphi_i\} = \dot{\varphi}_i \longrightarrow \varphi_i(t) = \varphi_i(0) + \omega_i t$$

$$\phi(x,t) = q_x(t) \to \infty$$

$x \to \infty$ $\quad q_i,\ i = 1 - N$



NSE

$$i \frac{\partial \psi}{\partial t} = -\frac{d^2 \psi}{d x^2} + 2g |\psi|^2 \psi$$

$$\updownarrow$$

$$Q_i(t) = Q_i(0) \qquad i = 1 \ldots \infty$$

Inverse scattering method

Lax pair : L, M

$$L = \frac{d}{dx} - \underset{2\times 2}{U} \quad , \quad M = \frac{d}{dt} - V$$

$$V = i \begin{pmatrix} \frac{\lambda^2}{2} + g|\psi|^2 & \sqrt{g}\left(-i\frac{d\psi^*}{dx} - \lambda\psi^*\right) \\ \sqrt{g}\left(-i\frac{d\psi}{dx} + \lambda\psi\right) & -\frac{\lambda^2}{2} - g|\psi|^2 \end{pmatrix}$$

$$U = i \begin{pmatrix} \frac{\lambda}{2} & \sqrt{g}\,\psi^* \\ -\sqrt{g}\,\psi & -\frac{\lambda}{2} \end{pmatrix}$$

$$[L, M] = -\frac{dV}{dx} + \frac{dU}{dt} + [U, V] = 0$$

① zero curvature condition for any λ

$$\underline{[L, M] = 0} \quad \overset{\text{eqs}}{\Longleftrightarrow} \quad \text{NLS}$$

$$\left[\frac{d}{dx}, f\right] g = \frac{d}{dx}(fg) - f\frac{dg}{dx} = \frac{df}{dx}g + \cancel{f\frac{dg}{dx}} - \cancel{f\frac{dg}{dx}}$$

$$V = i\left(\begin{array}{c|c} \frac{\lambda^2}{2} + g|\psi|^2 & \sqrt{g}\left(-i\frac{d\psi^*}{dx} - \lambda\psi^*\right) \\ \hline \sqrt{g}\left(-i\frac{d\psi}{dx} + \lambda\psi\right) & -\frac{\lambda^2}{2} - g|\psi|^2 \end{array}\right)$$

$$U = i\begin{pmatrix} \frac{\lambda}{2} & \sqrt{g}\,\psi^* \\ -\sqrt{g}\,\psi & -\frac{\lambda}{2} \end{pmatrix}$$

$$UV = -\begin{pmatrix} \frac{\lambda^3}{4} + \frac{g\lambda}{2}|\psi|^2 + g(-i\psi' + \lambda\psi)\psi^* & \sqrt{g}\,\frac{\lambda}{2}(-i\psi^{*\prime} - \lambda\psi^*) - \frac{\lambda^2}{2}\sqrt{g}\,\psi^* - g\sqrt{g}|\psi|^2\psi^* \\ \dots & \dots \end{pmatrix}$$

$$VU = -\begin{pmatrix} \frac{\lambda^3}{2} + g\frac{\lambda}{2}|\psi|^2 - g(-i\psi^{*\prime} - \lambda\psi^*)\psi & \dots \\ & \end{pmatrix}$$

$$[U,V] = \begin{pmatrix} g(i\psi' + \lambda\psi)\psi^* + g(i\psi^{*\prime} + \lambda\psi^*)\psi & \dots \\ \dots & \dots \end{pmatrix}$$

$$\frac{\partial U}{\partial t} = \begin{pmatrix} 0 & \dots \\ \dots & \end{pmatrix} \qquad \frac{\partial V}{\partial x} = i\begin{pmatrix} g(\psi'\psi^* + \psi^{*\prime}\psi) & \dots \\ \dots & \end{pmatrix}$$

② Lax.

$$\underset{\sim\sim}{\overrightarrow{\exp}}\left[\int U\,dx + V\,dt\right]$$

$$e^{x} \to e^{\begin{pmatrix}0&1\\1&0\end{pmatrix}} = \mathbb{1} + \begin{pmatrix}0&1\\1&0\end{pmatrix} + \frac{\mathbb{1}}{2} + \frac{\begin{pmatrix}0&1\\1&0\end{pmatrix}}{3!} + \cdots$$

$$\sum_{n=0}^{\infty}\frac{x^n}{n!} \rightsquigarrow e^{M} = \sum_{n=0}^{\infty}\frac{M^n}{n!} = \sum_{n=0}^{\infty}\frac{1}{(2n)!}\cdot\mathbb{1} + \sum_{n=0}^{\infty}\frac{1}{(2n+1)!}\begin{pmatrix}0&1\\1&0\end{pmatrix}$$

$$= \cosh(1) + \sinh(1)\cdot\begin{pmatrix}0&1\\1&0\end{pmatrix} = \begin{pmatrix}\cosh 1 & \sinh 1\\ \sinh 1 & \cosh 1\end{pmatrix}$$

$$\left(\det(\underbrace{e^{M}}_{N}) = e^{\operatorname{tr} M} \quad M = \log N \rightsquigarrow \det N = e^{\operatorname{tr}(\log N)} \quad \log(\det N) = \operatorname{tr}(\log N)\right)$$

A, B

$$\underbrace{e^{A+B} \neq e^{A} e^{B}}$$

Baker-Campbell-Hausdorff formula

$$e^{A+B} = e^{A + \frac{1}{2}[A,B] + \cdots + \frac{1}{n}[A,[A,\cdots[A,B]]] + \cdots} e^{B}$$

If $[A,B] = c \cdot \mathbb{1}$
scalar

$\to 0$

$$e^{A+B} = e^{A} e^{B} e^{[A,B]}$$

$\varepsilon(N-1)=1$

---

path ordered.

$x_1\ \varepsilon\ x_2 \qquad x_N$

$0 \qquad 1$

$x_N = 1$

$x_1 = 0$

$x_2 = \varepsilon$

$x_3 = 2\varepsilon$

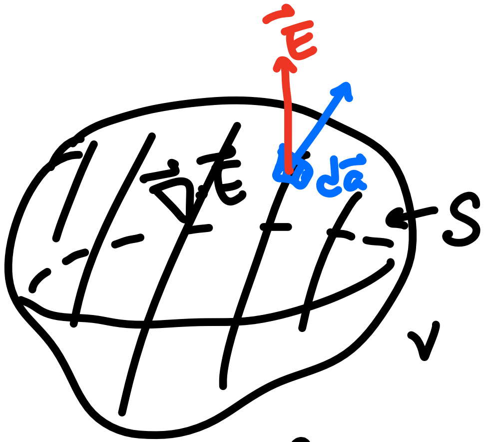
$\vdots$

$$\overrightarrow{e}^{\int_0^1 U\, dx} = \overrightarrow{e}^{\sum_{i=1}^{N} U(x_i) \cdot \varepsilon}$$

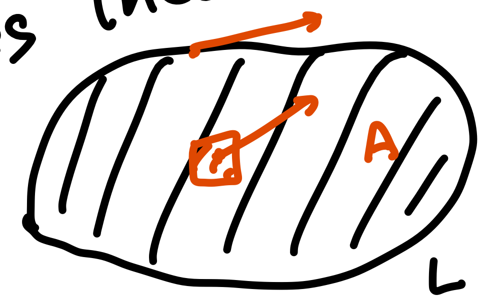
$$\equiv e^{\varepsilon U(x_1)} \cdot e^{\varepsilon U(x_2)} \cdots e^{\varepsilon U(x_N)}$$

Gauss law

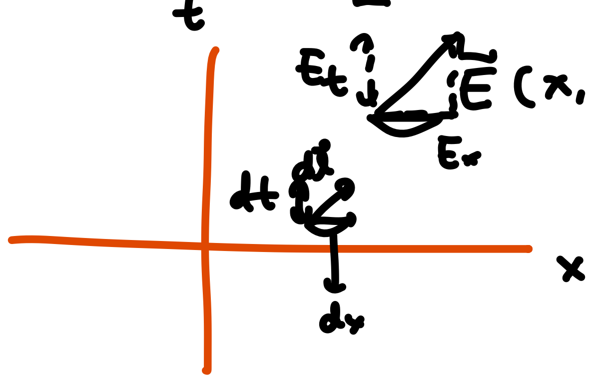
$$\int_V \vec{\nabla}\cdot\vec{E}\, d^3x = \oint_S \vec{E}\cdot d\vec{a}$$



Stokes theorem

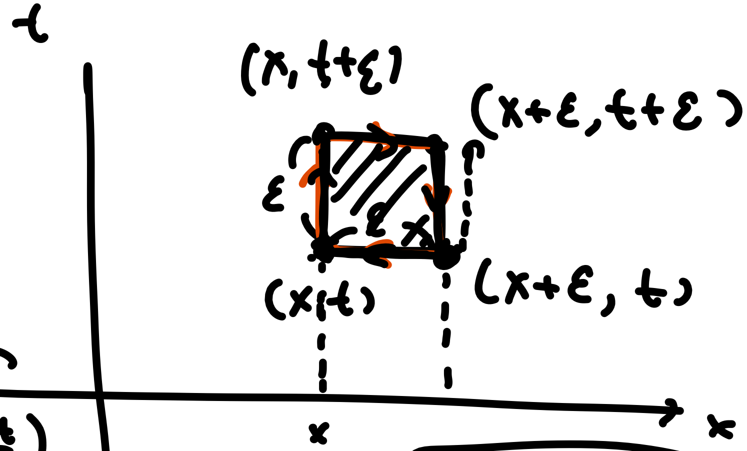


$$\int_A (\vec{\nabla}\times\vec{E})\cdot d\vec{a} = \oint_L \vec{E}\cdot d\vec{l}$$



$$\vec{E}(x,t) = (E_x, E_t)$$

$$\vec{E}\cdot d\vec{l} = E_x dx + E_t dt$$

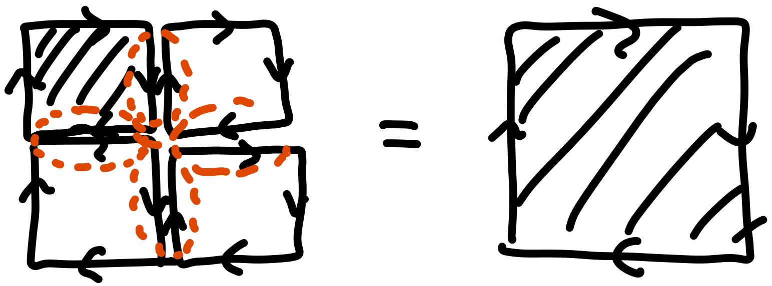


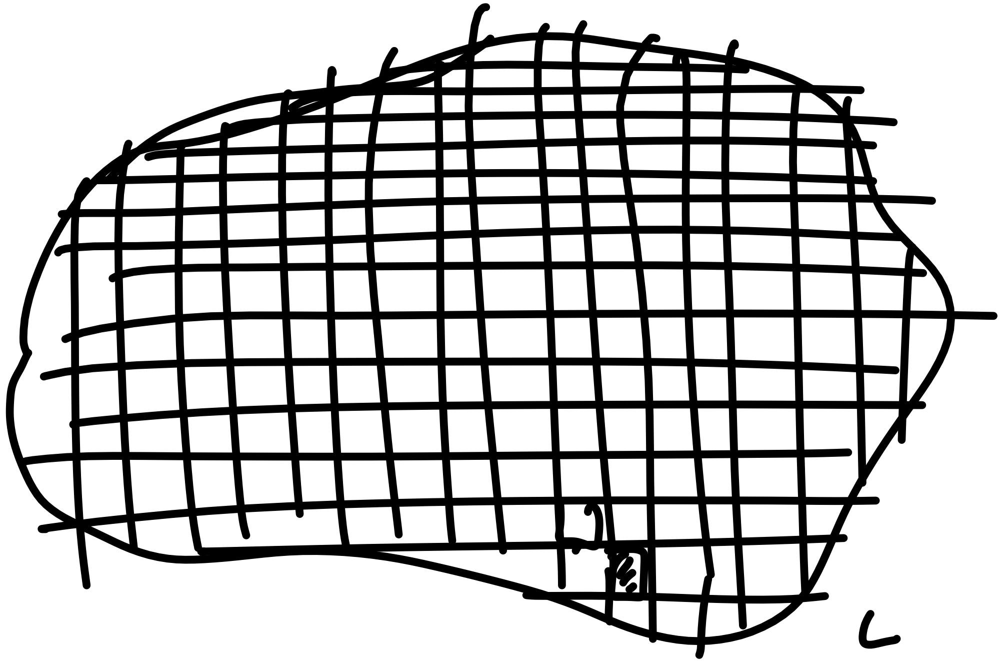
$$\oint_{\square} \vec{E}\cdot d\vec{l} = \oint E_x dx + E_t dt$$

$$= E_t(x,t)\,\varepsilon + E_x(x,t+\varepsilon)\,\varepsilon - E_t(x+\varepsilon,t)\,\varepsilon - E_x(x,t)\,\varepsilon$$

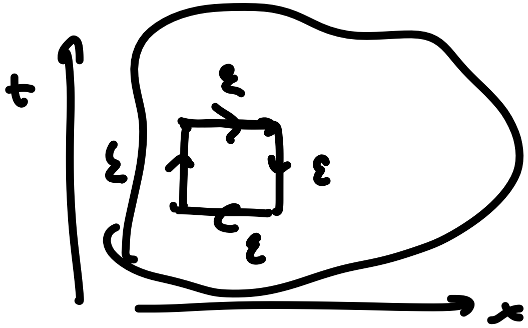
$$= \varepsilon\left(\underbrace{E_t(x,t) - E_t(x+\varepsilon,t)}_{-\frac{\partial E_t}{\partial x}\cdot\varepsilon}\right) + \varepsilon\left(\underbrace{E_x(x,t+\varepsilon) - E_x(x,t)}_{\varepsilon\frac{\partial}{\partial t}E_x}\right)$$

$$\boxed{\varepsilon^2}\left(\frac{\partial E_x}{\partial t} - \frac{\partial E_t}{\partial x}\right)$$





$$[L, M] = 0$$



$$\overrightarrow{\exp} \oint_{\square} U\,dx + V\,dt = \mathbb{1} \quad \text{if} \quad [L, M] = 0$$

$$e^{\varepsilon V(x,t)}\, e^{\varepsilon U(x,t+\varepsilon)}\, e^{-\varepsilon V(x+\varepsilon,t)}\, e^{-\varepsilon U(x,t)}$$

$$[U, V] \neq 0$$

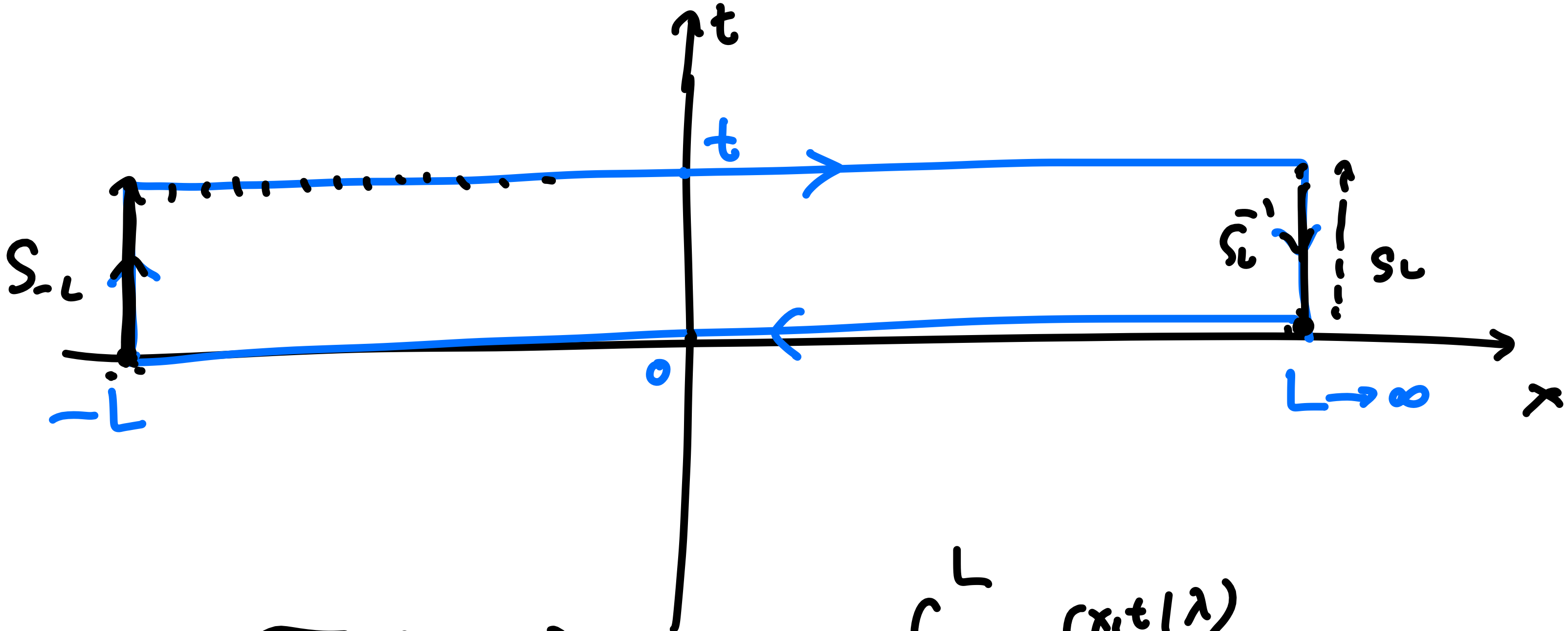
$$\boxed{\overrightarrow{\exp} \oint_{L} (U\,dx + V\,dt) = \mathbb{1}}$$

$$-\varepsilon \frac{\partial V}{\partial x}$$

$$e^{\varepsilon \left(V(x,t) + U(x,t+\varepsilon) - V(x+\varepsilon,t) - U(x,t) \right) + \frac{2\varepsilon^2}{2} [U,V]_{(x,t)}}$$

$$= e^{\varepsilon^2 \left(\frac{\partial U}{\partial t} - \frac{\partial V}{\partial x} + [U,V] \right)} = \mathbb{1}.$$

$$\underbrace{}_{=0}$$



$$T_L(\lambda,t) \equiv \overrightarrow{\exp}\int_{-L}^{L} U(x,t|\lambda)\,dx$$

$$U(L)=U(-L)$$
$$V(L)=V(-L)$$

$$S_L(\lambda) = \overrightarrow{\exp}\int_0^t V(\overset{L}{x},t|\lambda)\,dt \to S_L = S_{-L}$$

$L\to\infty$ peridicity : $\psi(L,t) = \psi(-L,t)$, $\psi^*(L,t)=\psi^*(-L,t)$

$$\mathbb{1} = S_{+L}\,\underset{\sim\!\sim\!\sim}{T_L(\lambda,t)}\,S_L^{-1}\,T_L^{-1}(\lambda,0)$$

$$\Updownarrow$$

$$T_L(\lambda,t) = S_L^{-1}\,T_L(\lambda,0)\,S_L$$

$$\underset{2\times 2}{T_L(\lambda,t)} = S_L^{-1}\, \underset{2\times 2}{T_L(\lambda,0)}\, S_L$$

$$\mathrm{tr}(AB) = \mathrm{tr}(BA)$$

$$\underset{\displaystyle \underset{\|}{} \; Q(t)}{\mathrm{tr}\, T_L(\lambda,t)} = \mathrm{tr}\big(\underbrace{S_L^{-1} T_L(\lambda,0)}_{A}\, \underbrace{S_L}_{B}\big) = \mathrm{tr}\big(\underbrace{S_L S_L^{-1}}_{\mathbb{1}} T_L(\lambda,0)\big)$$

"spectral parameter"

$$= \mathrm{tr}\big(T_L(\lambda,0)\big) = Q(0)$$

$$\mathrm{tr}\, T_L(\lambda,t) = P(\lambda,t) = \sum_{n=0}^{\infty} \underbrace{P_n(t)}\, \lambda^{n}$$

$n = 0, \ldots, \infty$

$\therefore$ $P_n(t)$ 는 모두 보존 전하!

무한개.