$$\tan\frac{\phi_{13}}{4} = \underbrace{\frac{a_1+a_3}{a_1-a_3}}\,\frac{\sinh\frac{y_1-y_3}{2}}{\cosh\frac{y_1+y_3}{2}}$$

$$a_3 = \frac{1}{a_1}$$

$$y_i = \frac{a_i+\frac{1}{a_i}}{2}\,x + \frac{a_i-\frac{1}{a_i}}{2}\,t$$

$$\frac{a_1+\frac{1}{a_1}}{a_1-\frac{1}{a_1}} = -\frac{1}{v}$$

$$y_1+y_3 = \left(a_1+\frac{1}{a_1}\right)\cdot x$$

$$y_1-y_3 = \left(a_1-\frac{1}{a_1}\right) t$$

$$\tan\frac{\phi}{4} = \frac{1}{v}\,\frac{\mathrm{sh}\left[\left(\frac{\frac{1}{a}-a}{2}\right)t\right]}{\mathrm{ch}\left[\left(\frac{a+\frac{1}{a}}{2}\right)x\right]}$$

$$\frac{\frac{1}{a}-a}{a+\frac{1}{a}} = v \qquad 2\gamma$$

$$\frac{1}{a}-a = 2\gamma v$$

$$\frac{1}{a}+a = 2\gamma$$

$$a = \gamma(1-v) = \frac{\sqrt{1-v}}{\sqrt{1+v}}$$

$$= \frac{1}{v}\,\frac{\mathrm{sh}(\gamma v t)}{\mathrm{ch}(\gamma x)}$$

$$\mathrm{sh}\,a = \frac{e^{a}-e^{-a}}{2}$$

$t \to -\infty$:
$$\approx \frac{1}{v}\,\frac{\left(-\frac{1}{2}\right)e^{-\gamma v t}}{\frac{1}{2}\,e^{-\gamma x}}$$

$a \to -\infty \quad e^{a} \lll e^{-a}$

$$\mathrm{sh}\,a \approx -\frac{e^{-a}}{2}$$

$x \to -\infty$

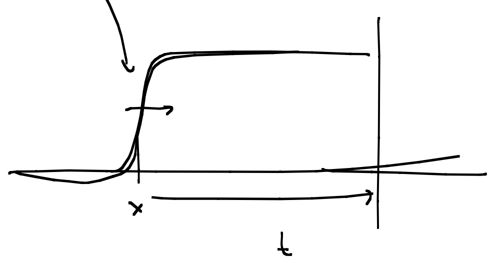
$$= -\frac{1}{v}\,e^{+\gamma(x-vt)} = -e^{\gamma\underbrace{(x-vt-\frac{1}{\gamma}\log v)}}$$

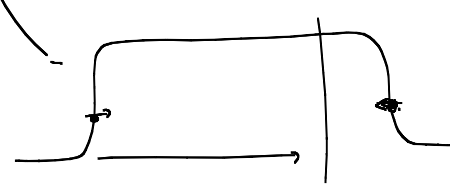
$$e^{\gamma\left(x-v\left(t+\underbrace{\frac{1}{\gamma v}\log v}_{\Delta t}\right)\right)}$$

<u>one-soliton</u>

$$\tan\frac{\phi}{4} = e^{\gamma(x-vt)}$$

$v<1 \qquad \underline{\Delta t<0}$





attractive

$$\text{time delay} = \frac{1}{\gamma v}\log_e v$$

$$v = \frac{\frac{1}{a}-a}{\frac{1}{a}+a} \qquad \gamma = \frac{a+\frac{1}{a}}{2}$$

$$= \frac{2}{\frac{1}{a}-a}\log\left(\quad\right)$$

---

soliton : $(1+1)$-d field theory $\quad \phi(x,t)$

time space

$(1+D)$-d field theory $\quad \phi(\vec{x},t)$

$D=2$ Vortex

$D=3$ Monopole $\leftarrow$ $\vec{\phi}(\vec{x},t)$ Hedge-Hog (고슴도치)

Derrick's Theorem.
$$E = \int d^D x\,\mathcal{H} = \int\Big[\underbrace{\tfrac{1}{2}\left(\cancel{\dot{\phi}^2}+(\vec{\nabla}\phi)^2\right)}_{H_1} + \underbrace{V(\phi)}_{H_2}\Big]\,d^D x$$

$$\vec{\nabla} = \left(\frac{\partial}{\partial x_1},\ \ldots,\ \frac{\partial}{\partial x_D}\right)$$

$$\phi(\vec{x},t) = \phi_0(\vec{x})$$

ex) $D=1$
$$\phi_0 = 4\tan^{-1}(e^{y})$$

$$\downarrow$$

$$\phi_\lambda(\vec{x}) = \phi_0(\lambda\vec{x})$$

$$E(\lambda) = \int_{-\infty}^{\infty}\left[\frac{1}{2}(\vec{\nabla}\phi_\lambda)^2 + V(\phi_\lambda)\right]d^D x$$

$\boxed{M\ \gamma}$

$$\phi_\lambda(\vec{x}) = \phi_0(\lambda\vec{x})$$

$$\Big(\frac{\lambda\partial}{\underbrace{\partial x_1}_{x_1'}}\phi_0(\underbrace{\lambda\vec{x}}_{\vec{x}'})\Big)^2 + \cdots + \Big(\frac{\lambda\partial}{\underbrace{\partial\lambda x_D}_{x_D'}}\phi_0\Big)^2$$

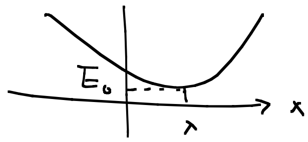
$$= \underbrace{\lambda^2\int \frac{1}{2}\left(\vec{\nabla}'\phi_0(\vec{x}')\right)^2 \frac{d^D(\overbrace{x\lambda}^{\vec{x}'})}{\lambda^D}}_{\lambda^{2-D}H_1} + \int V(\phi_0(\overbrace{\lambda\vec{x}}^{\vec{x}'}))\frac{d^D\lambda x}{\lambda^D}$$

$$\phi = \underline{\underline{\phi_0}}$$

$$+\ \lambda^{-D}H_2$$

$$\therefore\quad E(\lambda) = \lambda^{2-D}H_1 + \lambda^{-D}H_2$$

$\lambda = 1$ $\qquad E'(\lambda)\Big|_{\lambda=1} = 0$



$$\boxed{(2-D)H_1 - DH_2 = 0}$$

$$H_1 = \int \frac{1}{2}(\vec{\nabla}\phi_0)^2\, d^Dx \ \geqq 0$$

$$H_2 = \int V(\phi_0)\, d^Dx \ \geqq 0$$

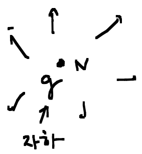
$\therefore$ ① $D > 2$; $\rightarrow$ $H_1 = H_2 = 0$

② $D = 2$: $\qquad H_2 = 0$

③ $D = 1$: $\qquad H_1 = H_2 = \int \frac{1}{2}\phi_0'^2\, dx = \int U(\phi_0)\, dx$

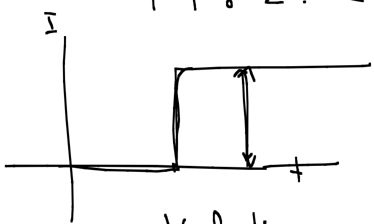
$$\phi_0' = \sqrt{2U} \ \rightarrow\ \frac{\phi_0'^2}{2} = U(\phi_0)$$

$$\vec{A}(\vec{x}) \propto \vec{x}\,\phi(\vec{x}) \longrightarrow \quad \vec{E} = -\vec{\nabla}\phi$$
$$\vec{B} = \vec{\nabla}\times\vec{A}$$

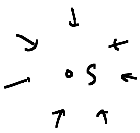


자기단극자
monopole

1982. 2.14



Valentine
monopole.

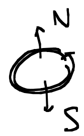


anti-monopole

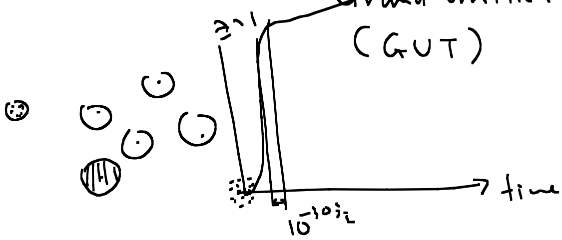
't Hooft – Polyakov
monopole.

$\boxed{B}\boxed{g} = \hbar \cdot n$

Dirac quantization



대통일이론
Grand Unified Theory
(GUT)



양성자 × 10^{30}

Big Bang → Monopole problem.
↓
Inflation theory